On the Optimal Reform of Income Support for Single Parents

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and

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Abstract

We characterize the optimal reform of U.S. income support for low-income single parents. We develop a heterogeneous agents model with idiosyncratic risk and incomplete asset markets where single parents evolve through three life stages defined by their children's care needs. Using the U.S. tax-transfer system as the benchmark policy and a sample of single mothers drawn from the CPS, we assess reforms that maximize the expected utility of entering mothers. When policy cannot be tagged by the single mothers' life stage, the optimal reform calls for an increase in out-of-work income support by 11 percent, from $6,320 to $7,080, and a decrease in the wage subsidy to low-wage workers from 34 to 22 percent. This reform delivers substantial welfare gains for single mothers-to-be, and has the support of a vast majority of incumbent mothers. Tagging policy by the life stage makes the government's trade-off between providing insurance to single mothers in stage one (child in pre-schooling age) and incentivizing them to work when they transit to stage two (child in school age) more favorable, thus increasing their scope for smoothing marginal utility throughout life stages. Single mothers in stage one receive $8,950 in out-of-work support, and no subsidies to low-wages. For single mothers in stage two the optimal reform prescribes a reduction in out-of-work income support and an increase in work subsidies. Tagging brings additional welfare gains.

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1 Introduction

In most developed economies, income support for low-income families with children consists of a mix of a guaranteed minimum income and earnings subsidies. In the U.S., the Temporary Assistance for Needy Families program (TANF) and the Supplemental Nutrition Assistance Program (SNAP) provide a guaranteed income (out-of-work income support) to eligible families with children that is phased out as their income increases.[1] On the other hand, the Earned Income Tax Credit (EITC), and to a lesser extent the Child Tax Credit (CTC), provide earnings subsidies to eligible workers. This paper is concerned with the optimal reform of income support for low-income, single-parent families, with special attention to the optimal mix of a guaranteed minimum income and earnings subsidies. Specifically, we take the U.S. tax-transfer system as a benchmark—accounting for all the non-convexities in taxes and income support—and then obtain the optimal reform that maximizes the ex-ante expected welfare of a single parent-to-be. We find that the optimal reform implies substantial welfare gains for entering single parents. We also find wide support for the adoption of the optimal policy among already existing single parents. To evaluate the welfare implications for these latter parents we characterize the transitional dynamics after the implementation of the reform. Our main motivation to focus on single-parent families is their higher at-risk-of-poverty rate, compared to two-parent families. For instance, in 2018 the poverty rate for single-mother families was 34 percent, against 6 percent for married families. Also, single mothers represent more than 90 percent of all the families receiving TANF cash benefits, and 60 percent of SNAP households with children (U.S. Census Bureau). The over-representation of single mothers among the poor and among those on welfare warrants an assessment of the existing anti-poverty policy for this demographic group.[2]

To carry out our analysis we use a dynamic structural model of consumption/savings and labor supply for single parents with one child. Labor supply includes both a participation decision (there are fixed costs of work), and the intensity of work (number of hours). In order to account for the dynamics and uncertainty inherent in the child's aging process, single parents in our model are assumed to evolve through three life stages. The first stage spans from childbirth up to the time the child enters school (which implies a change in market child care needs for working parents); the second stage ends when the child becomes ineligible as a dependent, where stage three begins. While,

[1] While SNAP provides food benefits and not cash payments to eligible families, we follow most of the literature in considering this in-kind assistance as income.

[2] It should also be noted that a two-parent family's behavioral response to policy involves trade-offs that are not available to single-parent families. For instance, two-parent families engage in within-household risk sharing, and adjust family labor supply to individual earnings shocks (the added worker effect). We hence follow a growing literature focusing on single parents (a review of key contributions to this literature is provided below).

on average, child care needs while working are higher in stage one (child not yet enrolled in formal schooling), single parents face idiosyncratic risk about these needs. They also face idiosyncratic earnings risk, both from shocks to their labor productivity and, in a model's extension, from human capital accumulation through learning by doing. Single parents can partially self-insure against all these risks by saving in a risk-free asset, and by managing labor supply (e.g. working longer hours when their labor productivity is high and their child care needs are low). Precautionary savings play an essential role in our results. First, the asset limits imposed by income support programs distort the saving decisions of low-income parents with assets close to those limits, yielding bunching in asset holdings. This results in less self-insurance, thus shaping optimal income support, particularly the optimal mix of guaranteed income and earnings subsidies. Second, the ability to self-insure through savings increases over the life cycle. A single parent in stage one is, on average, younger and, hence, less likely to have built a buffer stock of savings to use as self-insurance, compared to a parent that is already in stage two. This differential in the access to self-insurance through savings over the life cycle also shapes the optimal mix of guaranteed income and earnings subsidies.

We calibrate our model to the tax-transfer treatment of single parents with one child in the U.S., and to match key moments from a sample of non-college educated single mothers of one child drawn from the Annual Social and Economic Supplement of the Current Population Survey. Our calibrated model implies labor supply elasticities that differ both across the two margins of response and across the life cycle. Extensive margin elasticities are higher than intensive margin elasticities (0.78 versus 0.10); also, single mothers of a child in pre-schooling age are more responsive, on both margins, than single mothers of a school-age child.

The calibrated model is used to characterize the optimal reform of income support for single parents with one child. We restrict our analysis to reforms within the same parametric family of the current programs, subject to a government budget constraint. Specifically, we impose that the amount of income redistributed from the rest of the economy to the population of single mothers with one child is kept fixed. The restriction to a parametric family imposes no apparent constraints on the set of feasible reforms. That is, we can assess reforms with arbitrary profiles of marginal and participation tax rates, earnings subsidy rates, and levels of guaranteed income.

We find that the guaranteed minimum income under the current tax-transfer system is too low. Without tagging by the age of the child, the optimal reform prescribes an increase in the guaranteed income by 11 percent from its current value. By contrast, the EITC subsidy rate for low-earners is reduced by 12 percentage points. As a result, marginal and especially participation tax rates increase with the optimal reform, both of them becoming positive even at very low levels of earnings. The prescribed shift in the policy mix toward a higher out-of-work income support unveils the inefficiency of the current policy mix at smoothing single mothers' marginal utility across the life cycle. Single mothers in life stage one are more likely to be credit constrained and to face higher child care needs if they work. By increasing the guaranteed income, the optimal reform allows these mothers to ease credit constraints, thus improving their ability to smooth marginal utility across life stages. When we allow for tagging by age of child, the optimal policy for single mothers in stage one prescribes an increase in guaranteed income and no earnings subsidies. However, single mothers in stage two get a reduction in guaranteed income but receive earnings subsidies, thus incentivizing them to work. That is, the optimal reform under tagging increases further the scope for intertemporal marginal utility smoothing: Insurance in life stage one, and incentives to work in life stage two. Finally, we also consider an extension of our model with human capital accumulation by learning by doing. In this version of the model a new force shaping the optimal policy reform comes into play. Namely, since human capital accumulation has long-lasting effects on labor productivity and wages, there are positive social benefits from incentivizing single mothers in stage one to work (and hence to accumulate human capital). This tension between increasing insurance to single mothers in stage one while incentivizing them to work and accumulate human capital is resolved in the optimal reform by increasing both the guaranteed income and the earnings subsidy rate in the phase-in region. However, in order to meet the fixed-redistribution restriction, the phase-in region of the earnings subsidy is substantially shorter than that of the current policy.

Related literature. Our work in this paper is related to a growing literature on the optimal design of the tax system in economies where households are subject to idiosyncratic risk, and face incomplete asset markets and borrowing constraints. Since progressive taxation provides income insurance at the cost of distorting savings and labor supply, these models contain the key margins for a normative analysis of the tax code. In an influential paper, Conesa and Krueger (2006) find that the optimal earnings tax code is roughly a flat rate with a fixed deduction. Krueger and Ludwig (2016) extend the analysis by introducing a trade-off between tax progressivity and education subsidies. They find that the optimal tax code is far less progressive, becoming close to a proportional system. Heathcote et al. (2017) study optimal tax progressivity in a framework that allows for analytical solutions. In a version of their model where poverty constrains investment in skills, optimal progressivity is close to the U.S. value. Karabarbounis (2016) studies age-dependent optimal earnings taxation and finds that tax distortions should be hump shaped in age. Heathcote et al. (2020) allow the progressivity of the tax code to depend on age and find that (1) progressivity should be U-shaped in age, and (2) the average marginal tax rate should be increasing and concave in age. Our work in this paper departs from the above-mentioned studies in two important ways. First, while their focus is on the joint characterization of the optimal earnings tax code for all households across the earnings distribution, our focus is restricted to low-income households, taking the tax code for high incomes as given. We fix the level of redistribution from high-income to low-income households and do not characterize the optimal tax code for all income levels jointly. Second, in the above-mentioned papers the tax-transfer system is approximated using a smooth function of earnings that depends on three parameters. The optimal system is then found by choosing the values of these parameters that maximize a welfare criterion. By contrast, in this paper we model taxes and transfers as they actually are, with all their kinks and non-convexities. Then, we search for the optimal system within this parametric family. Beyond the value of this approach in terms of readiness of the results for practical implementation, it has two other main advantages. On the one hand, using a smooth function to approximate net taxes greatly reduces the nature and scope of the reforms that can be considered. Since our focus is only on the optimal tax-transfer system to low-income households, by adopting the parametric family of the current system we retain the ability to assess the optimality of thresholds and the phasing in and out of transfers. On the other hand, the smooth functions used in the literature to approximate net taxes are not defined at zero earnings. However, since our population of interest is made up of non-educated single mothers with one dependent child—a population that can face potentially high child care costs if they work—it is important to assess tax-transfer systems that imply zero earnings for a non-negligible fraction of households, thus rendering the use of these smooth approximations less suitable for our purposes.

Our work is also related to Blundell and Shephard (2012), Mullins (2019) and Ho and Pavoni (2020). Blundell and Shephard are concerned with the optimal taxation of earnings of single mothers of one child. They develop a static model of labor supply featuring both unobserved heterogeneity in preferences and child care costs that depend on household's observable characteristics. Their model consequently abstracts from a consumption/savings decision and from the dynamics introduced by child aging. They use the UK tax-transfer schedule as the policy environment and then assess reforms consisting of a level of out-of-work income support and nine different marginal tax rates. The optimal schedule is characterized by an out-of-work income support of 130 pounds per week (in prices of 2002) and by a non-monotonic profile of marginal tax rates. They also find a welfare improving role for tagging according to the age of the child. Mullins is also concerned with the design of income transfers to single mothers. He develops a dynamic model of labor supply that includes maternal investment in the child's skill development. His model, however, abstracts both from investment in financial assets—and hence from the self-insurance role of savings—and from child care costs. By restricting to the family of continuous, one-kink policies, Mullins finds that optimal transfers to single mothers of one child are characterized by an out-of-work income support of 97 dollars per week (in prices of 2010), which is then phased out. Ho and Pavoni are interested in the role of private information in child care policy. They use a Mirrleesian model of single mothers' child care and labor supply decisions where their labor productivity is private information. Assuming that all mothers face the same child care needs, the authors characterize the child care subsidies and income transfers that implement the constrained-efficient allocation. They find positive subsidies to formal child care, with subsidies declining in mother's earnings. Finally, our study in this paper also relates to and builds upon recent positive analyses on taxes and female labor supply. For instance, using a life-cycle model where individuals face no risks after entering the economy, Guner et al. (2020) compare the effects on female labor supply of expansions in childcare credits versus expansions in child credits. They find that the former lead to a large increase in married female labor market participation, while the latter have a negative effect. Both expansions, however, generate welfare gains for newborn households. (For other related studies see Kaygusuz 2010, Guner et al. 2012, and Ortigueira and Siassi 2021, to name a few.)

The remainder of the paper is organized as follows. Section 2 presents our structural model of consumption/savings and labor supply for the analysis of optimal transfers. The data, the calibration of the model, and the benchmark solution are presented in Section 3. Section 4 contains the analysis of the optimal reform of the tax-transfer system, both without and with tagging by child age. Section 5 presents an extension of our model that includes returns to experience. Concluding remarks are offered in Section 6. There are three appendices.

2 The Model

Demographics. Our population of interest is made up by non-college educated, never-married single mothers with one dependent child. In our model, a single mother lives through three different stages, $\chi \in \{1, 2, 3\}$, defined by the child's care needs and qualification as dependent. The first stage, $\chi = 1$, spans the period before the child enters formal education, a period in which he/she needs full-time care. Children are assumed to "age" stochastically, i.e. with per-period probability m_{12} the child moves to the second stage of childhood ($\chi = 2$). This stage spans the period starting when the child enters formal education, and ending when the child becomes ineligible as a dependent. Our modeling of child aging as a stochastic process implies that the biological age of the child and the age at which child care needs change are not perfectly coupled, introducing uncertainty about when this change will take place. Examples of this type of uncertainty are the potential availability of employer-provided child care, or admission in pre-school education. (Admission to the Head Start Program—which helps children from disadvantaged populations to better succeed in school—is limited, and seats are assigned via a lottery process.) On the other hand, illness or disability might prolong the need for all-day child care beyond the age of school entry.

Children in stage two ($\chi = 2$) become ineligible as dependents with per-period probability

m_{23}, upon which they leave the household and their mothers remain childless ($\chi = 3$). Again, uncertainty about when a child in stage two becomes ineligible as a dependent may stem from a number of sources. For instance, children that marry before turning 18 can no longer be claimed as dependents; by contrast children who continue education after turning 18 remain eligible as dependent children.

More compactly, a single mother transits through these three life stages according to a Markov chain with transition matrix

$$M = \begin{pmatrix} m_{11} & m_{12} & 0 \\ 0 & m_{22} & m_{23} \\ 0 & 0 & 1 \end{pmatrix}. \tag{2.1}$$

To keep our population of interest (single mothers in stages one and two) constant and normalized to one, every period a mass $q = m_{12}m_{23}/(m_{12} + m_{23})$ of single mothers enter the economy in stage one. This yields a stationary mass of $m_{23}/(m_{12} + m_{23})$ single mothers in stage one, and a stationary mass of $m_{12}/(m_{12} + m_{23})$ single mothers in stage two.

Preferences. Preferences are described by a per-period utility function, $U(c/n_\chi, h)$, and by a discount factor β. Household consumption is denoted by c; and the equivalence scale parameters, n_χ for $\chi = 1, 2, 3$, account for the change in size and composition of the household (single mothers in stage three are childless). The number of hours allocated to work are denoted by h. Remaining hours, $1 - h$, are allocated to non-market activities, which may include leisure, time spent with the child, etc. We do not model the split of non-working hours across these alternative uses, and, hence, our utility function should be interpreted as aggregating the mother's utility from the time devoted to all these uses.

Labor productivity, earnings, income, and assets. The productivity of time devoted to market work depends on a deterministic, stage-specific component, ε_χ, as well as on an idiosyncratic stochastic component, z. Then, a single mother that supplies h hours to market work receives labor income $e \equiv \varepsilon_\chi z h w$ before taxes and transfer payments, where w is the wage per efficiency unit of labor. When we extend the model to include human capital accumulation, the stage-specific component ε_χ will be replaced by a process of learning-by-doing, according to which the allocation of hours to market work can increase productivity.

The idiosyncratic stochastic component, z, is assumed to evolve according to the process

$$\ln z' = \rho \ln z + \epsilon, \qquad \text{with } \epsilon \sim N(0, \sigma_\epsilon^2), \tag{2.2}$$

where ϵ is an idiosyncratic shock. Entering single mothers draw the initial idiosyncratic productivity level, z_0, from the log-normal distribution $LN\big(0, \sigma_\epsilon/(1 - \rho^2)\big)$. Single mothers enter the economy without assets, face a borrowing constraint, but can save in a one-period risk-free bond. Asset holdings are denoted by a, and the capital income derived from these assets by ra, where r is the risk-free interest rate. All single mothers in stages one and two also receive child support, ϑ, from their child's father. Hence, pre-tax-and-transfer income is $\varepsilon_\chi zhw + ra + \vartheta$. (Note that child support is not considered as income by the IRS.)

Child care costs. Single mothers in stages one and two may incur child care costs while working. Child care costs are assumed to depend on the child's life stage (χ), on the number of hours worked by the mother (h), and on idiosyncratic characteristics of the mother (η). We think of the latter as characteristics determined by the mother's social network, which may provide a number of hours of free child care when the mother is at work (family members that do not live in the household, friends, neighbors, a church, etc.). For instance, a neighbor takes care of the child for, say, one hour a day; or the child is with her father a number of hours per week. Entering single mothers draw a value for η from a log-normal distribution with parameters μ_1^η and σ_1^η. The value of η remains unchanged until the mother enters stage two, when she draws a new value from a log-normal distribution with parameters μ_2^η and σ_2^η. We denote the working mothers' child care cost function by $\Gamma_\chi(h, \eta)$.

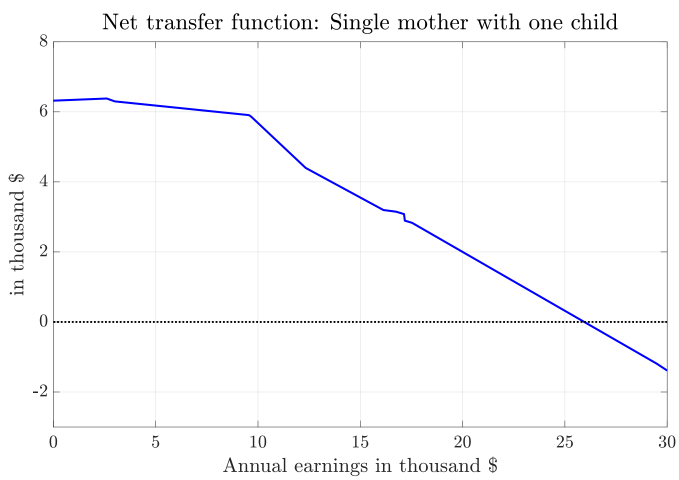
Taxes and transfers. Our model embeds the following tax-transfer programs: individual income and payroll taxes, the Earned Income Tax Credit (EITC), the Child Tax Credit (CTC), Temporary Assistance for Needy Families (TANF), and the Supplemental Nutrition Assistance Program (SNAP). We model these tax and transfer programs as they are in the U.S., including all their kinks and non-convexities. Specifically, the income tax scheme in the model contains the seven tax brackets, the tax deduction, and the personal exemptions. The payroll tax is a flat rate with a tax cap. The EITC is refundable, and the CTC has two tranches, one refundable and one non-refundable. TANF and SNAP are assistance programs that provide a guaranteed income to eligible applicants. We include the actual eligibility criteria, tax credit rates, out-of-work income support, and phase-outs.[3] Appendix A presents detailed descriptions of these tax and transfer programs, and provides an account of the non-convexities created by each program on the budget constraint of single mothers. We denote the net transfer received by a single mother (tax credits and assistance transfers minus income and payroll taxes paid) by $TT_\chi(a, e, h, \eta)$, that is

[3] We abstract from stigma from participation in income programs and from any other friction that could prevent participation. Available estimates of take-up rates are obtained from the entire U.S population of eligible households. While we are not aware of estimates for take-up rates for the demographic group considered in our study, it can be argued that they are higher than those estimated for the entire population. For this reason, we focus on the effects of the statutory rules governing income programs and abstract from stigma or any other friction. Additionally, we also abstract from TANF time limits since families with children can receive child-only TANF assistance beyond the 60 months threshold.

$$TT_{\chi}(a,e,h,\eta) = \underbrace{\Big(EITC^{j}(a,e) + CTC^{j}(a,e) + TANF(a,e,h,\eta) + SNAP^{\jmath}(a,e,h,\eta)\Big)}_{\text{tax credits and assistance transfers}} - \underbrace{\Big(T^{j}(a,e) + T_{p}(e)\Big)}_{\text{income and payroll taxes}}.$$

The dependence of the net transfer function on χ stems from: (i) the filling status, j (mothers in stages one and two file as head of household with one dependent, and mothers in stage three file as single); (ii) mothers in stage three are not eligible for TANF, and (iii) SNAP eligibility and benefits depend on the type of assistance unit, $\jmath$. The dependency on h and η stems from the child care deductions used in the calculation of net income for the purpose of determining TANF and SNAP eligibility and benefits (see Appendix A for details). As an illustration, Figure 1 displays the net transfer function for single mothers with no assets and no child care costs.

FIGURE 1– NET TRANSFER FUNCTION



Bellman equations. We now write down the problems solved by single mothers in our economy. Since single mothers evolve stochastically through three life stages, we write their problems as follows.

The maximization problem of single mothers in life stage one. Single mothers enter the economy in stage one ($\chi = 1$) with no assets, draw a productivity level z_0, and a child care cost parameter η. The state vector of a single mother in life stage one is (z, a, η). We denote her value function by

$v_1(z,a,\eta)$, which is defined as

$$v_1(z,a,\eta) = \max_{c,h,a'} \left\{ U(c/n_1,h) + \beta \left(m_{11}\,\mathbb{E}\left[v_1(z',a',\eta)|z\right] + m_{12}\,\mathbb{E}\left[v_2(z',a',\eta')|z\right] \right) \right\} \tag{2.3}$$

$$\begin{aligned} \text{s.t.} \quad & c + \Gamma_1(h,\eta) + a' = e + (1+r)a + \vartheta + TT_1(a,e,h,\eta) \\ & z' \text{ is given by the stochastic process (2.2)} \\ & \eta' \text{ is drawn from probability distribution } LN(\mu_2^\eta, \sigma_2^\eta) \\ & 0 \le h \le 1 \quad \text{and} \quad a' \in [0,\bar{a}], \end{aligned}$$

where $e = \varepsilon_1 zhw$; z_0 is drawn from the probability distribution $LN\big(0, \sigma_\epsilon/(1-\rho^2)\big)$; η is drawn from $LN(\mu_1^\eta, \sigma_1^\eta)$; $a_0 = 0$; and $v_2(z,a,\eta)$ is the value function of single mothers in life stage two. The set of asset holdings, $[0,\bar{a}]$, makes explicit the assumption of a borrowing constraint ($\bar{a}$ is a non-binding upper bound).

The maximization problem of single mothers in life stage two. Upon entering stage two ($\chi = 2$), single mothers draw a new child care cost parameter, η, and solve the following problem

$$v_2(z,a,\eta) = \max_{c,h,a'} \left\{ U(c/n_2,h) + \beta \left(m_{22}\,\mathbb{E}\left[v_2(z',a',\eta)|z\right] + m_{23}\,\mathbb{E}\left[v_3(z',a')|z\right] \right) \right\} \tag{2.4}$$

$$\begin{aligned} \text{s.t.} \quad & c + \Gamma_2(h,\eta) + a' = e + (1+r)a + \vartheta + TT_2(a,e,h,\eta) \\ & z' \text{ is given by the stochastic process (2.2)} \\ & 0 \le h \le 1 \quad \text{and} \quad a' \in [0,\bar{a}], \end{aligned}$$

where $e = \varepsilon_2 zhw$, and $v_3(z,a)$ is the value function of single mothers in stage three. Although our focus is on the optimal reform of the tax-transfer system to single mothers with one dependent child (stages one and two), we include stage three in the model so that we get a continuation value for mothers in stage two. As will be made clearer below, in the calibrated model single mothers transit to stage three at age 45, on average, and, hence, still face more than two decades of labor market participation. Mothers in stage three do not need to pay child care costs while working, $\Gamma_3(h,\eta) = 0$, do not receive child support, $\vartheta = 0$, and solve the problem $v_3(z,a) = \max_{c,h,a'} \{U(c,h) + \beta\mathbb{E}\big[v_3(z',a')|z\big]\}$, subject to the budget constraint $c + a' = e + (1+r)a + TT_3(a,e,h,\eta)$, and to the borrowing constraint. Note that the equivalence scale for households in stage three is equal to one since the adult child has left the household. Note also that the tax-transfer function of these mothers does not depend on h and η since they are not eligible for TANF and do not pay child care while working. We assume that mothers in this stage are subject to the same productivity shocks as in stages one and two.

It should be emphasized that the tax-transfer system to single mothers in stage three (i.e. without dependents) will be kept fixed in our analysis of the optimal reforms. Additionally, we will conduct our analysis of the optimal reforms both without tagging by the age of child—i.e., by restricting $TT_1(a, e, h, \eta) = TT_2(a, e, h, \eta)$—and with tagging by age, where the income transfers are allowed to depend on the age of the child (the life stage χ).

2.1 Parameterization

Preferences. Per-period utility is represented by the standard, additively separable utility function in consumption and non-market time, extended to include a (semi-)fixed cost of labor market participation. This participation cost is assumed to take on three different values, depending on whether the single mother chooses to work part time (PT), full time (FT) , or extra time (ET). We adopt the following functional form

$$U(c/n_\chi, h) = \frac{(c/n_\chi)^{1-\sigma} - 1}{1-\sigma} + \varphi \frac{(1-h)^{1-\zeta} - 1}{1-\zeta} - \mathbb{1}_{h>0} \times \begin{cases} \nu_{PT} & \text{if } 0 < h \leq \bar{h}_1 \\ \nu_{FT} & \text{if } \bar{h}_1 < h \leq \bar{h}_2 \\ \nu_{ET} & \text{if } h > \bar{h}_2, \end{cases} \tag{2.5}$$

where σ is the coefficient of relative risk aversion, φ is a utility weight on non-market time, and $\zeta > 0$ controls the Frisch elasticity of labor supply. The last term in (2.5) is the utility cost of labor market participation, reflecting psychological and other non-economic costs such as being unable to share meals with the child, or missing the child's wake up and bedtime hours. We will calibrate the parameters ν_{PT}, ν_{FT}, ν_{ET}, and set the thresholds $\bar{h}_1$ and $\bar{h}_2$ to match the relative shares of working mothers across part-, full- and extra-time employment. Part-time work corresponds to working less than or equal to 1,050 hours per year; full-time work corresponds to working between 1,051 and 2,100 hour; and extra-time work to more than 2,100 hours.

Child care costs. The child care cost function is parameterized as follows

$$\Gamma_\chi(h, \eta) = \max\left\{\gamma_\chi + \eta \times h \ , \ 0\right\} \qquad \text{for} \quad \chi = 1, 2, \tag{2.6}$$

where γ_1, $\gamma_2 < 0$ are parameters. As already stated, parameter η is drawn from log-normal distributions $\eta \sim LN(\mu^\eta_\chi, \sigma^\eta_\chi)$, where μ^η_χ and σ^η_χ are allowed to depend on the life stage $\chi = 1, 2$. Note that the idiosyncratic component η determines (i) the number of hours worked at which the mother needs to start purchasing child care, and (ii) the cost of child care per hour. To the left of the intercept with the hours-axis, child care costs are zero (e.g., because someone from the mother's social network can look after her child for a few hours a day), thus rationalizing the empirical observation that many working single mothers do not pay child care while working. At the same time, the model delivers heterogeneity in paid child care costs across mothers with the same number of hours worked, which is also in accordance with what we observe in the data.

3 Data, Calibration and Model Fit

3.1 Data and Summary Statistics

We now describe the sample selection criteria. Our sample of single mothers with one dependent child is drawn from the 2012, 2013, 2014, 2015 and 2016 Annual Social and Economic (ASEC) Supplements of the CPS. A single mother with one dependent child is in our sample if she meets all the following conditions: (1) She lives with her child who is under 19 years of age; (2) She has never been married and is under 65 years of age; (3) The father of the child does not live with them in the same dwelling; (4) The single mother has no family member, other than her child, living in the same dwelling; (5) Her highest educational degree is a high school diploma; (6) She is not in the armed forces; (7) She did not receive any income from: business and/or farm activities, disability, retirement, social security, unemployment, veterans income, and survivors' benefits. Our final sample of single mothers in life stages one and two has $2,323$ households. Table 3 (columns [1], [3] and [5]) below presents summary statistics of labor market variables (employment rates, annual hours worked, and earnings) for the single mothers in our sample. Earnings are expressed in dollars of 2013. Single mothers of one child under 5 ($\chi = 1$) have a lower employment rate than single mothers of one child between 5 and 18 ($\chi = 2$): 75.7 percent versus 80.8 percent. Conditional on working, they also supply less hours to work: $1,550$ annual hours versus $1,779$. Average annual earnings, conditional on working, are also significantly lower among single mothers with one child under 5: \$18,713 versus \$26,440. In addition to the lower hours worked by single mothers of one child under 5, the difference in annual earnings is also explained by lower hourly earnings: \$12.01 versus \$14.67.

3.2 Calibration

Parameters calibrated exogenously. A period in the model is one year. The transition probabilities between the different life stages are set at $m_{12} = 1/5$ and $m_{23} = 1/13$, so that on average a child spends 5 years in stage one and 13 years in stage two. For the coefficient of relative risk aversion we choose a standard value of $\sigma = 1.5$: this is exactly the midpoint value of the interval estimated by Chetty (2006). The real interest rate is set at $r = 0.02$. Using the average child support paid to single mothers in our sample we set $\vartheta = 3,000$.[4] The two parameters

[4]While there is dispersion in child support received by single mothers in our sample, the model abstracts from both heterogeneity and risk in child support. There are several reasons for this modeling choice. It is not uncommon that custodian parents have informal agreements with noncustodial parents concerning child support payments which may lead to misreporting. Also, liquidity-constrained noncustodial parents that missed payments may pay out more than one year of child support in a single installment. Since child support risk for lone mothers is not well understood

characterizing the labor productivity process are set as in Flodén and Lindé (2001), who estimate $\rho = 0.914$ and $\sigma_\epsilon = 0.206$.[5] Entering mothers draw their initial productivity level from the ergodic distribution implied by this autoregressive process. We apply the equivalence scales used by the CPS to calculate per-adult equivalent household expenses and set $n_1 = n_2 = (1 + 0.8)^{0.7} = 1.509$ and $n_3 = 1$. The parameter ζ in the utility function affects the Frisch elasticity of labor supply. In models like ours where budget constraints contain kinks and preferences are discontinuous (fixed costs of labor market participation), this parameter is difficult to pin down. For this reason, we first set ζ to 3 so that the Frisch elasticity of a mother that works a number of hours equal to the average in our sample and that her choice is not in a kink equals 0.77. As a sensitivity analysis we then increase ζ to 4, which is associated with a Frisch elasticity of 0.58. Table 1 presents the parameter values set outside of the model in our benchmark economy.

TABLE 1– PARAMETERS SET EXOGENOUSLY

Description	Parameter	Value	Description	Parameter	Value
Transition probability	m_{12}	1/5	Real interest rate	r	0.020
Transition probability	m_{23}	1/13	Equivalence scale	n_1	1.509
Risk aversion	σ	1.5	Equivalence scale	n_2	1.509
Curv. non-market time	ζ	3	Equivalence scale	n_3	1.000
Hours threshold	$\bar{h}_1$	0.1918	Labor productivity	ρ	0.914
Hours threshold	$\bar{h}_2$	0.3836	Labor productivity	σ_ϵ	0.206
Child support	ϑ	\$3,000			

The values for the parameters in the tax-transfer programs correspond to those in fiscal year 2013 (see Appendix B for a presentation of these parameters and a description of the sources).

Parameters calibrated endogenously. The remaining 13 parameters are set so that the model matches the following 13 empirical moment conditions (we report in parenthesis the parameter that influences each moment the most):

1. Average hours worked, conditional on working, represent 30.8 percent of the time endowment. (φ)

2-4. The labor market participation rate is 78.6 percent. The fractions of working mothers who work part-time, full-time and extra-time are 19.9 percent, 73.1 percent and 7.1 percent respectively. ($\nu_{PT}, \nu_{FT}, \nu_{ET}$)

5-6. Average hourly earnings of mothers in stage one are \$12.01. The corresponding value for

we set ϑ to its mean value for all lone mothers.

[5] We conducted a sensitivity analysis with respect to parameter σ_ϵ and found no substantially different results.

mothers in stage two is \$14.67 (w, ε_2).

7-8. Average child care expenditures, conditional on being positive, paid by working mothers of one child under 5 are \$3,072. The corresponding value for mothers of one child between 5 and 18 is \$2,764. (μ_1^η, μ_2^η)

9-10. The standard deviation of child care expenditures, conditional on being positive, paid by mothers of one child under 5 are \$2,564. The corresponding value for mothers of one child between 5 and 18 is \$2,182. $(\sigma_1^\eta, \sigma_2^\eta)$

11-12. The fraction of mothers of one child under 5 that pay child care costs while working is 41.4 percent. The corresponding value for mothers of a child between 5 and 18 is 21.2 percent. (γ_1, γ_2)

13. Average wealth among non-college educated single mothers of one child is \$29,132 (Survey of Consumer Finances 2016). (β)

Table 2 reports the parameter values that match these moments.

TABLE 2– PARAMETERS CALIBRATED ENDOGENOUSLY

Description	Param.	Value	Moment	Target	Model
Weight non-market time	φ	0.1255	Avg hours worked	0.308	0.309
Participation cost PT	ν_{PT}	0.1330	Employment rate	0.786	0.784
Participation cost FT	ν_{FT}	0.1436	Fraction full-time	0.731	0.732
Participation cost ET	ν_{ET}	0.1477	Fraction extra-time	0.071	0.071
Wage rate	w	49.7	Hourly earnings $(\chi = 1)$	12.01	12.01
Stage-specific productivity	ε_2	1.31	Hourly earnings $(\chi = 2)$	14.67	14.69
Log-normal distribution	μ_1^η	1.81	Avg. child care paid	3,072	3,042
Log-normal distribution	μ_2^η	0.13	Avg. child care paid	2,764	2,713
Log-normal distribution	σ_1^η	2.28	Std. child care paid	2,564	2,533
Log-normal distribution	σ_2^η	2.82	Std. child care paid	2,182	2,181
Child care intercept	γ_1	-2.53	Frac. paid child care	0.414	0.414
Child care intercept	γ_2	-3.14	Frac. paid child care	0.212	0.213
Discount factor	β	0.9584	Avg. wealth	29,132	29,189

Notes: Parameters calibrated endogenously. Notice that the value of the deterministic component of labor productivity in stage one, ε_1, has been normalized to one.

3.3 Benchmark Solution and Model fit

Summary statistics for labor market outcomes obtained from the model are shown in Table 3 (columns [2], [4] and [6]). The model fits well moments that were not used as targets. The em-

TABLE 3– SUMMARY STATISTICS: DATA AND MODEL UNDER BENCHMARK POLICY

	Single mothers of one child aged 0-18		Single mothers of one child under 5		Single mothers of one child aged 5-18	
	[1] Data	[2] Model	[3] Data	[4] Model	[5] Data	[6] Model
Employment rate (%)	78.6[a]	78.4[a]	75.7	72.1	80.8	80.8
Part-time (%)	19.9[a]	19.7[a]	27.0	30.8	15.3	15.9
Full-time (%)	73.1[a]	73.2[a]	67.7	67.1	76.4	75.4
Extra-time (%)	7.1[a]	7.1[a]	5.3	2.2	8.3	8.7
Annual hours worked*						
Average	1,688[a]	1,690[a]	1,550	1,523	1,779	1,747
Std.dev.	635	412	670	406	594	398
Median	2,040	1,792	1,820	1,670	2,080	1,833
p25	1,300	1,343	1,040	1,044	1,560	1,540
p75	2,080	1,955	2,080	1,812	2,080	1,996
Annual earnings* ($)						
Average	23,363	25,280	18,713	19,050	26,440	27,418
Std.dev.	21,805	17.326	15,336	12,456	24,650	18,223
Median	20,000	19,568	15,781	16,458	22,396	21,696
p25	11,434	10,299	8,865	9,676	13,792	13,653
p75	30,000	31,946	24,943	22,783	33,257	35,148
Hourly earnings* ($)						
Average	13.61	14.01	12.01[a]	12.01[a]	14.69[a]	14.69[a]
Std.dev.	11.65	6.69	9.88	5.14	12.54	7.02
Median	11.45	12.49	10.06	10.91	12.31	13.08
p25	8.11	9.05	7.14	8.28	8.95	9.48
p75	16.34	17.23	14.20	14.38	17.78	18.04

Notes: Summary statistics for our sample of single mothers, and from the model under the benchmark policy. [a]Moments used as targets in the calibration. *Conditional on working.

ployment rates of mothers in stages one and two match well those in the data. The distributions of annual hours worked, total earnings, and hourly earnings are also close to their empirical values. For instance, the first quartiles of hours worked for mothers in stages one and two in the model are, respectively, \$1,044 and \$1,560, versus \$1,040 and \$1,540 in the data. The third quartiles are also close to their empirical values: \$1,812 and \$1,996 in the model versus \$2,080 and \$2,080 in the data. The model yields standard deviations which are somewhat lower than in the data, mainly because the model does not generate enough mothers at the far end of the two tails of the hours distribution. While some authors choose to remove observations below, say, the fifth percentile and above the 95 percentile, our summary statistics in Table 3 correspond to the uncensored data.

The labor supply responses to changes in the wage rate generated by the model under our parameter values are presented in Table 4. We report the extensive, intensive, and total hours elasticities for different subsamples of single mothers and holding fixed the distribution of wealth, which implies that the reported elasticities should be interpreted as short-run elasticities. Extensive margin elasticities are computed as the percentage change in employment rates after a one percent change in wages. For the whole sample we obtain an extensive margin elasticity of 0.78; the elasticity is higher for mothers in stage one, 1.05; and lower for mothers in stage two, 0.68. Intensive margin elasticities are computed as the percentage change in hours worked among working mothers (i.e. those working both before and after the change in the wage rate). For the whole sample of mothers the intensive margin elasticity is 0.10; being higher for mothers in stage one, 0.46; and close to zero for mothers in stage two. Total hours elasticities are 0.93 for the whole sample, and 1.74 and 0.70 for mothers in stages one and two, respectively. In sum, under the benchmark policy and parameter values the participation margin is more responsive than the intensive margin. Also, the labor supply of mothers in stage one is more responsive than that of mothers in stage two. While there are no direct empirical counterparts against which these responses can be compared, they are in line with the simulated wage elasticities for single mothers obtained from the models used by other authors (e.g. Blundell and Shephard 2012, Blundell et al. 2016).

In the benchmark economy, total transfers paid to single mothers in stages one and two minus total tax revenues collected from them amount to \$1,448 per household. This corresponds to the amount of income redistributed from the rest of the economy—which is left unmodeled in this study—to our population of interest. As will become clearer in Section 4, we will assume throughout this study that the amount of redistribution towards this demographic group remains constant at \$1,448 (on a per household basis).

TABLE 4– ELASTICITIES OF LABOR SUPPLY FOR OUR SAMPLE OF SINGLE MOTHERS

	Single mothers of one child aged 0-18	Single mothers of one child under 5	Single mothers of one child aged 5-18
	[1]	[2]	[3]
Extensive margin	0.78	1.05	0.68
Intensive margin	0.10	0.46	0.01
Total hours	0.93	1.74	0.70

Notes: Elasticities of labor supply for single mothers with one child under the benchmark tax-transfer policy and calibrated parameter values.

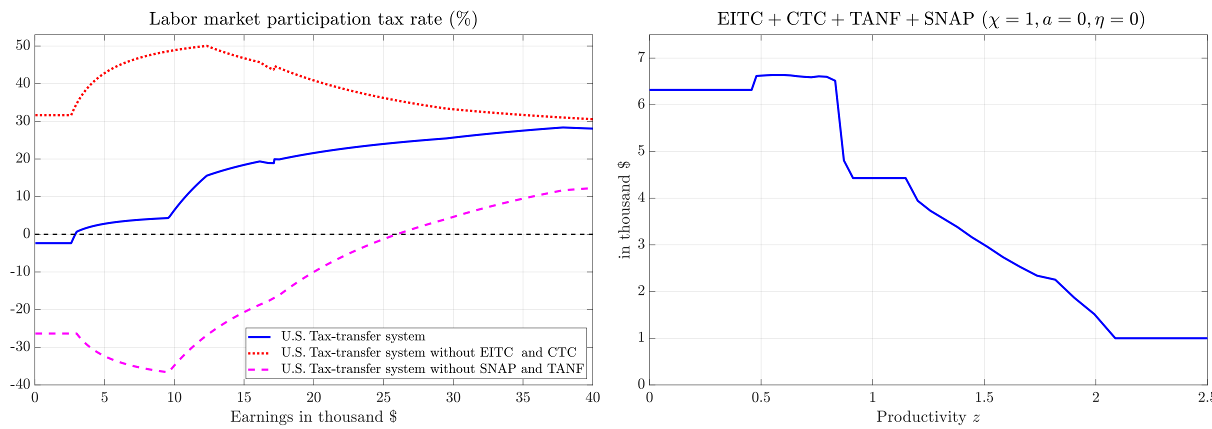
3.4 Guaranteed Income and Work Incentives in the U.S. Tax-Transfer System

Before moving to the analysis of the optimal reform of income support, we find it useful to start with an illustration of how the current tax-transfer system trades off a guaranteed income (TANF and SNAP) with subsidies to work (EITC and CTC). The left panel of Figure 2 plots the labor market participation tax rates at different levels of earnings that emerge from the current tax-transfer system (solid blue line), and from two counterfactuals. For the sake of expositional clarity, the participation tax rates in Figure 2 correspond to single mothers that do not need to pay child care while working and have no wealth. Participation tax rates are computed as the difference between net transfers if working and net transfers if not working, as a percentage of earnings

$$\tau^P(e) = -\frac{TT(e) - TT(0)}{e}. \tag{3.1}$$

As is apparent from Figure 2, current tax-transfer programs yield negative participation tax rates of about -2 percent at low levels of earnings, and then increase to reach values of about 27 percent at earnings over \$35,000. The average participation tax rate among working mothers in our benchmark economy is 9.2 percent. (Note that mothers with assets above the TANF and SNAP asset limit face lower rates than mothers with no assets.) Participation tax rates depend critically on the level of guaranteed income and how quickly it phases out, as well as on the subsidies to low-wage workers. It is useful to look at the two polar cases that emerge under no guaranteed income, on the one hand, and under no subsidies, on the other hand. The dashed purple line in Figure 2 shows the participation tax rates in the first case (i.e. if TANF and SNAP were to be removed). In this case, working mothers earning up to \$26,000 would face negative participation tax rates. In fact, single mothers earning between five and ten thousand dollars would get a participation subsidy amounting to more than one third of their earnings. The dotted red line shows the participation tax rates in the second polar case (i.e. if EITC and CTC were to be removed). Single mothers would

FIGURE 2– PARTICIPATION TAX RATES AND INCOME FROM EITC, CTC, TANF, SNAP



Notes: Left panel: Labor market participation tax rates for a single mother with no assets and no child care cots. Right panel: Sum of income collected by a single mother with no assets and no child care costs from the EITC, CTC, TANF and SNAP as a function of labor productivity.

face participation tax rates higher than 40 percent at earnings levels between \$4,000 and \$22,000, and higher than 30 percent up to earnings of \$40,000. The actual participation tax rates for single mothers with no assets lie roughly in the middle of those generated by these two polar cases.

We also use the policy function for hours worked obtained from the solution of the model to illustrate how the current mix of guaranteed income and work subsidies shapes the relationship between labor productivity, labor supply decisions, and income transfers. The right panel of Figure 2 displays the sum of income collected by single mothers from the EITC, CTC, TANF and SNAP as a function of their labor productivity. Specifically, we use the optimal policy of single mothers in life stage one with no assets and no child care costs while working. (Using the optimal policy function of mothers in life stage two would yield qualitatively similar results.) The flat segment at about \$6,400 (the guaranteed income) shows that mothers with labor productivity below 0.45—which is about 41 percent of the average productivity—choose not to work. Above this productivity threshold they supply hours to market work, and the sum of the transfers they collect from these programs does not decline monotonically with labor productivity. The mix of guaranteed income and work incentives affects both the productivity threshold for participation and the level of hours worked conditional on participation. In the next section we assess the optimality of this mix in the current U.S. tax-transfer system.

4 The Optimal Reform of the Tax-Transfer System

This section describes the quantitative exercises and presents our main findings characterizing the optimal reform of the tax-transfer system. As explained above, we assess reforms within the parametric family of the actual system. Since the tax-transfer system in our model includes two tax codes (income and payroll taxes), two tax credits (EITC and CTC) and two income welfare programs (TANF and SNAP), and since each contains a large number of parameters, we must necessarily specialize our analysis to a subset of these parameters. Notwithstanding this restriction, we retain enough flexibility to generate and assess a large set of feasible reforms that fully reshape the current system, especially in terms of the generated trade-off between a guaranteed income and work incentives. The reforms assessed in this paper also maintain the asset-eligibility conditions of the current system. Specifically, we retain the asset-income limit of the EITC, and the asset limits of TANF and SNAP.

The set of feasible reforms. For the sake of clarity, let us re-write the EITC equation for eligible workers as (see Appendix A for details)

$$I(e) = \begin{cases} \kappa_0 + \kappa_1 e & \text{if} \quad 0 \leq e < \mathrm{e}_{I_1} \\ \kappa_0 + \kappa_1 \mathrm{e}_{I_1} & \text{if} \quad \mathrm{e}_{I_1} \leq e < \mathrm{e}_{I_2} \\ \max\{\kappa_0 + \kappa_1 \mathrm{e}_{I_1} - \kappa_2(e - \mathrm{e}_{I_2}), 0\} & \text{if} \quad e \geq \mathrm{e}_{I_2}. \end{cases}$$

By definition of an earned income tax credit, the value of κ_0 in the current EITC is equal to zero; κ_1 is the earnings subsidy rate in the phase-in region, and κ_2 is the phase-out rate. The thresholds, e_{I_1} and e_{I_2}, mark the end of the phase-in region and the beginning of the phase-out region, respectively. In the region between these two thresholds, the credit plateaus at its maximum value $\kappa_1 \mathrm{e}_{I_1}$ (see Figure A1 in Appendix A).

We consider reforms that change the values of $(\kappa_0, \kappa_1, e_{I_1})$ and that are revenue neutral (i.e., the government's net transfer to the population of single mothers with one dependent child is kept constant at the level of the benchmark economy). We let the government maximize over (κ_0, κ_1), and adjust e_{I_1} so that revenue neutrality is attained. Note that with only these three parameters we can generate a continuum of reforms, spanning the following two extreme policies. On the one hand, the EITC can be transformed into a TANF-like program ($\kappa_0 > 0$ and $\kappa_1 = 0$), which would increase the guaranteed income and remove work subsidies. On the other hand, the sum of TANF, SNAP and EITC can be transformed into an EITC-like program ($\kappa_0 < 0$ and $\kappa_1 > 0$), which would remove the guaranteed income and increase work subsidies. We can thus assess income-transfer systems with arbitrarily positive or negative marginal tax rates at low income levels.

Welfare Criterion. We compute the reform of the transfer system that maximizes the ex-ante utility of an entering single mother. By ex ante we mean before the entering single mother draws her initial level of productivity, z, and idiosyncratic characteristic, η, which determines a number of hours of free child care in life stage one. The government's maximization problem is therefore written as

$$\max_{\kappa_0,\kappa_1,e_{I_1}} \int_z \int_\eta v_1(z, a=0, \eta; \kappa_0, \kappa_1, e_{I_1}) f(z) g(\eta) dz d\eta, \tag{4.1}$$

subject to the revenue-neutrality condition. Function $v_1(z, a=0, \eta; \kappa_0, \kappa_1, e_{I_1})$ is the value function of an entering single mother under policy parameters κ_0, κ_1 and e_{I_1}. Function $f(\cdot)$ is the density of the log-normal distribution with parameters μ_z and σ_z; function $g(\cdot)$ is the density of the log-normal distribution with parameters μ_1^η and σ_1^η.

In order to assess the support for the optimal reform among incumbent single mothers at the time the reform is introduced (i.e. those in our benchmark steady-state solution), we compute the solution of the model along the transition to the new steady state. We then assess the welfare effects of the optimal reform for the mothers in the benchmark solution.

4.1 The Optimal Reform without Tagging by Age of Child

We start with the case in which policy cannot depend on the age of the child. That is, the policy parameters that define the set of policy reforms, $(\kappa_0, \kappa_1, e_{I_1})$ are not allowed to vary across life stages: mothers in stages one and two are given equal treatment by the government. Under this restriction, the optimal policy must trade off the provision of insurance to single mothers in stage one against the provision of work incentives to mothers in stage two. Since mothers in stage one enter with no assets, face relatively higher child care needs if they work, and a lower market wage rate, they are more likely to be credit constrained than mothers in stage one. As working longer hours is the only means for credit-constrained mothers to smooth consumption, providing insurance via a higher guaranteed income allows these mothers to allocate labor more efficiently, especially across life stages, and smooth marginal utility. On the other hand, the provision of work incentives helps to retain mothers in stage two in the labor force, which contributes to alleviate the budget constraint of the government. The optimal level of guaranteed income, the subsidy rate to low earnings, and the length of the phase-in region that maximize the utility of an entering single mother are presented below.

The Welfare-maximizing Income-transfer System. The values of the three parameters (κ_0, κ_1, e_{I_1}) at the optimal transfer system are presented in column [2] of Table 5. Compared to the values of the benchmark policy, (\$0, 0.34, \$9.559), the optimal reform amounts to a partial

TABLE 5– BENCHMARK AND OPTIMAL POLICY

	Benchmark policy [1]	Optimal policy w/o tagging [2]	Optimal policy w. tagging [3]
κ_0			
$\chi = 1$	\$0	\$760	\$2,626
$\chi = 2$	\$0	\$760	−\$500
κ_1			
$\chi = 1$	34%	22%	0%
$\chi = 2$	34%	22%	18%
e_{I_1}			
$\chi = 1$	\$9,559	\$9,577	\$0
$\chi = 2$	\$9,559	\$9,577	\$17,530

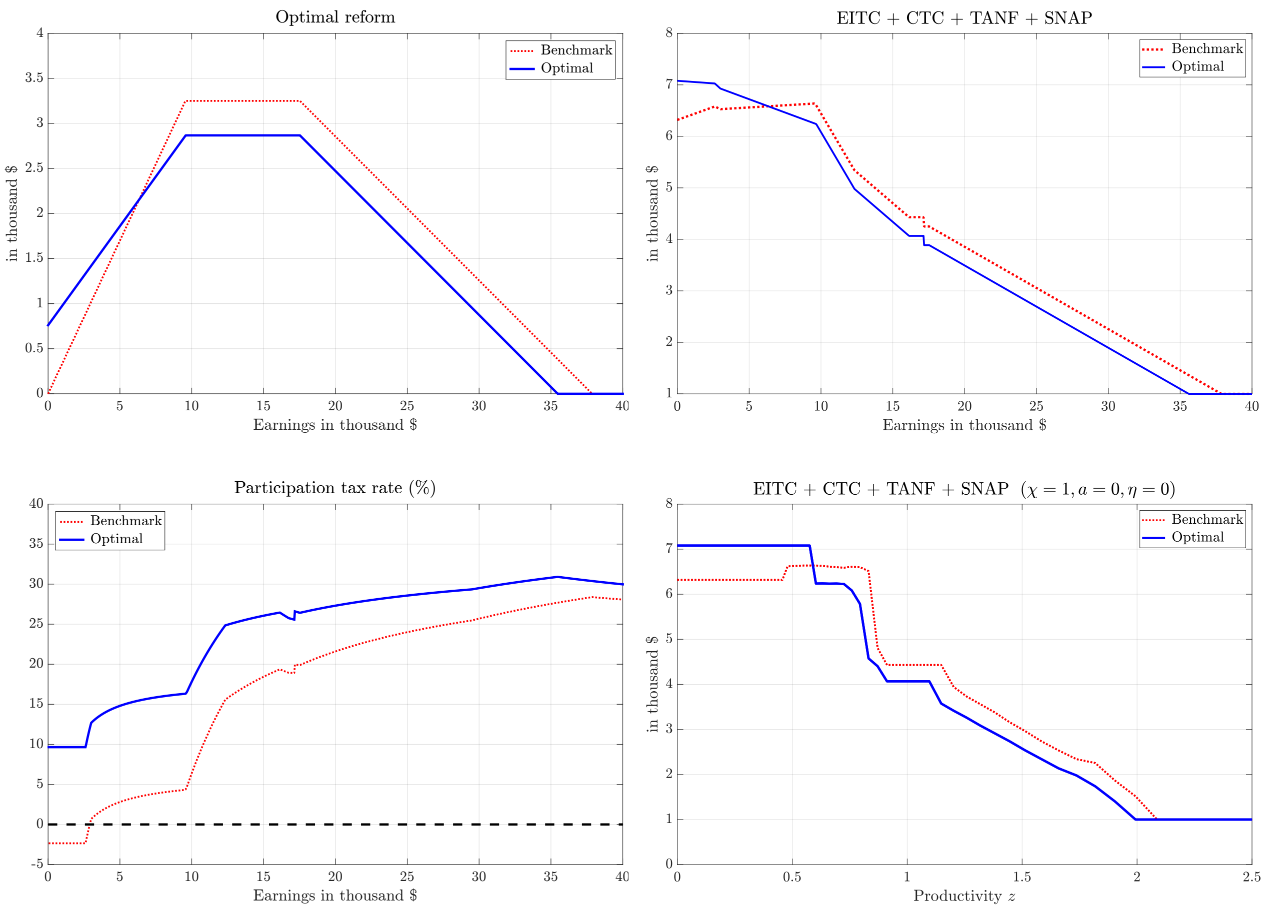
transformation of the EITC towards a TANF-like program. Namely, the value of κ_0 increases from \$0 to \$760, which, added to income from TANF and SNAP, yields an increase in the guaranteed income from \$6,320 to \$7,080.[6] The earnings subsidy rate in the phase-in region, κ_1, decreases from the current value of 0.34 to 0.22; and the income threshold defining the end of phase-in region, e_{I_1}, barely changes from \$9,559 to \$9,577, resulting in a lower tax credit in the now slightly shorter plateau region.

Figure 3 displays the benchmark and the optimal EITC schemes (top-left panel), and the sum of income transfers from the EITC, CTC, TANF and SNAP collected by a single mother with no assets as a function of her earnings (top-right panel). It is readily clear from this latter figure that the shift from work subsidies to the guaranteed income brought about by the optimal policy comes at the cost of positive marginal tax rates at low earnings levels. In contrast to the benchmark policy, the guaranteed income in the optimal reform is phased out from the first dollar of earnings. The shift toward more guaranteed income also increases labor market participation tax rates. The bottom-left panel of Figure 3 shows the participation tax rates faced by mothers with no assets and no child care costs. The optimal policy yields positive participation tax rates even at very low earnings, increasing actual tax rates by as much as 10 percentage points up to earnings of about \$15,000. As a result, labor supply drops significantly under the optimal policy, especially among

[6] Recall that the EITC has an investment income limit of \$3,300 but no asset limit for eligibility. Hence, the additional \$760 prescribed by the optimal policy are paid to all non-working mothers with investment income below \$3,300. Some of these mothers, however, may not be eligible for TANF and SNAP as they may fail the asset limit test for these latter programs. In short, the \$760 can be regarded as a child allowance to single mothers with no labor earnings and with investment income below \$3,300 per year.

low-productive mothers. The productivity threshold for labor market participation increases from 0.45 to 0.57; that is, from 41 to 52 percent of the average productivity. This is illustrated in the bottom-right panel of Figure 3, which plots total income collected from EITC, CTC, TANF and SNAP as a function of labor productivity. This panel also shows that under the optimal policy working mothers collect less income from these programs than they would under the benchmark.

FIGURE 3– BENCHMARK AND OPTIMAL POLICY WITHOUT TAGGING



Notes: Top-left panel: Benchmark and optimal EITC schemes. Top-right panel: Benchmark and optimal income transfers to single mothers with no assets as function of earnings. Bottom-left panel: Benchmark and optimal participation tax rates of single mothers with no assets. Bottom-right panel: Benchmark and optimal income transfers to single mothers with no assets as function of labor productivity.

Model summary statistics under the optimal policy are presented in column [3] of Table 6. In line with the discussion above, employment rates decline, especially among mothers in stage one (non-school-age child). Namely, the employment rate of these mothers declines from 72.1 percent under the current policy to 53 percent under the optimal; for mothers in stage two, employment falls from 80.8 to 68.4 percent. However, conditional on working, average hours worked increase.

This effect follows from the change in the composition of working mothers: Low-productive mothers drop from the labor market under the optimal policy, hence increasing average hours. Among the mothers that work both under the benchmark and the optimal policy, average hours worked remain mostly unchanged.

Average annual earnings, conditional on working, increase by about $3,000. Again, this is explained by the fact that the low-productive mothers that are pulled into the labor market by the high working subsidies of the benchmark policy leave the market under the optimal. (The average participation tax rate increases from 9.2 percent in the benchmark to 17.8 percent in the optimal.) The decomposition of total net transfers between working and non-working mothers reveals the relative shift away from working subsidies in the optimal policy. Working mothers go from being net transfer recipients under the benchmark policy to net tax payers under the optimal. When we decompose net transfers by life stage we find that both groups of mothers continue to be net transfer recipients under the optimal policy. It will become clearer below that this latter result will not hold when we allow for tagging, as the government will use policy to change the allocation of consumption and hours worked across life stages.

While overall average disposable income declines slightly, mothers in the first quintile gain, on average, about three hundred dollars in disposable income under the optimal policy. By contrast, mothers in the second quintile lose more than three thousand dollars. These are mostly low productive mothers that move from employment under the benchmark policy to non-employment under the optimal reform.

The optimal policy increases the efficiency of the allocation of hours across productivity realizations. The correlation between hours worked and labor productivity among mothers in stage one increases from 0.59 under the benchmark policy to 0.65 under the optimal. For mothers in stage two this correlation increases from 0.55 to 0.60. The increase is strikingly higher for single mothers in stage one with low assets holdings, who are more likely to face a binding credit constraint. For instance, the correlation between hours and labor productivity among mothers in stage one with assets below the median increases from 0.36 to 0.58. This is because the increased insurance of the optimal policy allows these mothers to smooth consumption without having to work long hours in periods of low labor productivity. To see this, average annual hours worked by mothers in stage one with no assets equals 975 hours under the benchmark, against 685 under the optimal policy.

The increase in insurance brought about by the higher guaranteed income of the optimal policy crowds out self-insurance, and, hence, reduces wealth accumulation. The percentage of mothers with no assets increases from 26.2 percent under the benchmark policy to 27.4 percent under the optimal. Likewise, the percentage of mothers holding less than two thousand dollars in wealth

TABLE 6– SUMMARY STATISTICS: BENCHMARK AND OPTIMAL POLICY

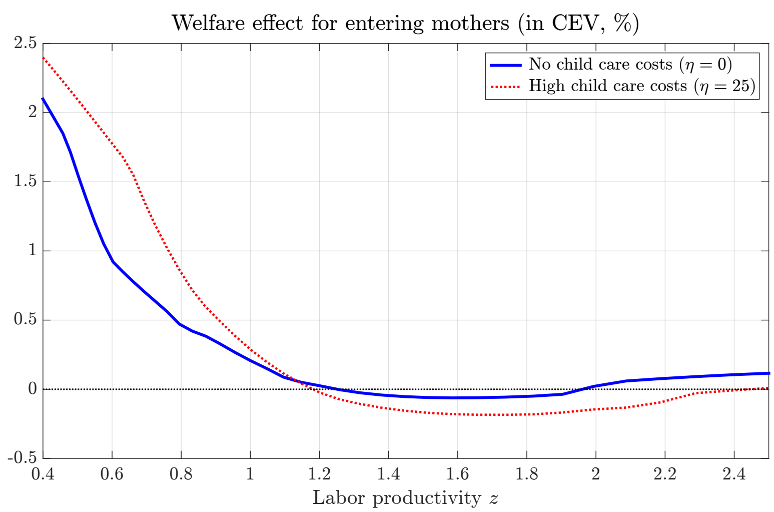
Description	Sample [1]	Benchmark policy [2]	Optimal policy w/o tagging [3]	Optimal policy w. tagging [4]
Employment rate (%)	All mothers	78.4	64.2	62.3
	$\chi = 1$	72.1	53.4	20.8
	$\chi = 2$	80.8	68.4	78.3
Part-time work (%)	All mothers	19.7	12.9	3.6
Full-time work (%)	All mothers	73.2	78.0	86.7
Extra-time work (%)	All mothers	7.1	9.1	9.7
Annual hours worked*	All mothers	1,690	1,799	1,908
	$\chi = 1$	1,523	1,644	1,963
	$\chi = 2$	1,747	1,846	1,903
Annual earnings* ($)	All mothers	25,280	28,769	29,974
	$\chi = 1$	19,050	22,614	34,776
	$\chi = 2$	27,418	30,617	29,483
Hourly earnings* ($)	All mothers	14.01	15.17	15.19
	$\chi = 1$	12.01	13.26	17.34
	$\chi = 2$	14.69	15.75	14.97
Total net transfer ($)	All mothers	1,448	1,448	1,447
	Working	511	−555	−1,002
	Non working	937	2,004	2,449
	$\chi = 1$	1,051	1,065	1,737
	$\chi = 2$	398	383	−290
Disposable income ($)	All mothers	24,257	23,007	23,244
Quintile 1		8,086	8,362	9,755
Quintile 2		15,092	11,622	12,346
Quintile 3		21,543	20,708	21,036
Quintile 4		28,180	27,762	27,708
Quintile 5		46,375	46,602	46,776
Corr(hours,productivity)	$\chi = 1$	0.60	0.65	0.71
	$\chi = 2$	0.56	0.60	0.50
Wealth ($)	All mothers	29,189	28,022	30,451
Frac. with no assets	All mothers	26.2	27.4	29.4
Frac. with ≤ $2,000	All mothers	61.8	63.7	61.3
Welfare	Entering mother	–	+ 0.63%	+ 1.73%

Notes: Model summary statistics under benchmark and optimal policy. *Conditional on working.

increases from 61.8 to 63.7 percent.

The Welfare Gain from the Optimal Reform and its Decomposition. The welfare gain for a single mother-to-be from entering the economy under the optimal policy, relative to entering under the benchmark, amounts to an increase by 0.63 percent in consumption in every period and state of nature. To get a better understanding of this consumption-equivalent welfare increase, we first look at the distribution of welfare gains across entering mothers upon drawing a productivity level (see Figure 4). As expected, mothers that draw a low productivity value win the most, especially if they also draw a high child care cost parameter, η, for the first life stage. Welfare gains become negative for productivity levels between 1.2 and 2.2 (i.e. 1.1 and 2 times the average productivity), and then approach zero for high-productive mothers. In summary, the welfare gain from our behind-the-veil-of-ignorance optimal policy stems mainly from the large gains accruing to low-productive entering mothers. The relatively low welfare losses accruing to more productive mothers are traded-off for insurance against low productivity and high child care cost realizations.

FIGURE 4– WELFARE EFFECTS FROM OPTIMAL REFORM



A decomposition of the aggregate welfare gain along the lines suggested in Conesa et al. (2009) pins down the contributions from changes in consumption and non-market time. Each of these two sources of welfare gain can in turn be decomposed into a level component and a volatility component.[7] The breakdown of the aggregate welfare gain from our reform across all these components (see Table 7) reveals that the gains stem from the increase in the average level of non-market time, and the increase in consumption smoothing across labor productivity and child care cost realizations. Note that the lower average consumption, and the higher dispersion of non-market

[7]See Appendix C for a detailed explanation of how all these components are calculated in our model.

time brought about by the reform contribute negatively to welfare. Also, the higher dispersion of non-market time follows in large part from the relatively higher decrease in employment during life stage one.

TABLE 7– DECOMPOSITION OF WELFARE GAIN

Total change		0.63
Consumption	Total	−3.22
	Level	−4.91
	Dispersion	1.78
Non-market time	Total	3.98
	Level	6.60
	Dispersion	−2.40

Notes: All numbers in percent.

Reform support among mothers in the benchmark solution. Using the model's transitional dynamics after the optimal policy reform is implemented we can compute the welfare effects for incumbent single mothers in stages one and two at the time of the implementation. We find that 82.4 percent of these mothers support the reform, against 17.6 percent who do not. The overall average welfare gain for these mothers amounts to an increase by 0.41 percent in consumption-equivalence terms. Mothers in the first life stage gain the most, with a welfare increase by 0.65 percent, compared to a 0.31 percent increase among mothers in the second life stage.

Among the single mothers that do not support the reform, most of them are in the second life stage and have medium-high labor productivity. For these mothers, the gains from the ex-post insurance brought forth by the reform are not big enough to offset the costs from the reduction in work subsidies. However, their welfare losses are small. Conditional on losing, the average welfare loss is −0.08 percent. It is apparent from these results that both entering single mothers and most incumbent mothers in the benchmark solution would support a reform of the current income-transfer system that helps them further their ability to smooth marginal utility, especially across life stages. A higher guaranteed income in conjunction with lower work subsidies enable single mothers-to-be and many of those in life stage one to shift utility from the second to the first stage of life.

Robustness with respect to the preference parameter ζ. The parameter that controls the curvature of the utility function with respect to non-market time is linked principally to the intensive-margin Frisch elasticity of labor supply. As discussed above, this parameter is difficult to pin down, especially in a model like ours with all the kinks and non-convexities of the tax-transfer system. To investigate the robustness of our results with respect to this parameter, we increase its value from 3 to 4, which would amount to a reduction in the Frisch elasticity from 0.77 to 0.58.[8] All other parameters are consequently re-calibrated so that the model continues to match our list of empirical moment conditions. The optimal policy reform now yields the following values: ($\kappa_0 = \$770$, $\kappa_1 = 0.22$, $e_{I_1} = \$9,328$), which are very close to the ones obtained under the benchmark calibration. The guaranteed income prescribed by the optimal reform is now $7,090, instead of $7,100, and the wage subsidy to low-wage earners remains unchanged at 22 percent. Aggregate variables are also robust to changes in the value of ζ.

4.2 The Optimal Reform with Tagging by Age of Child

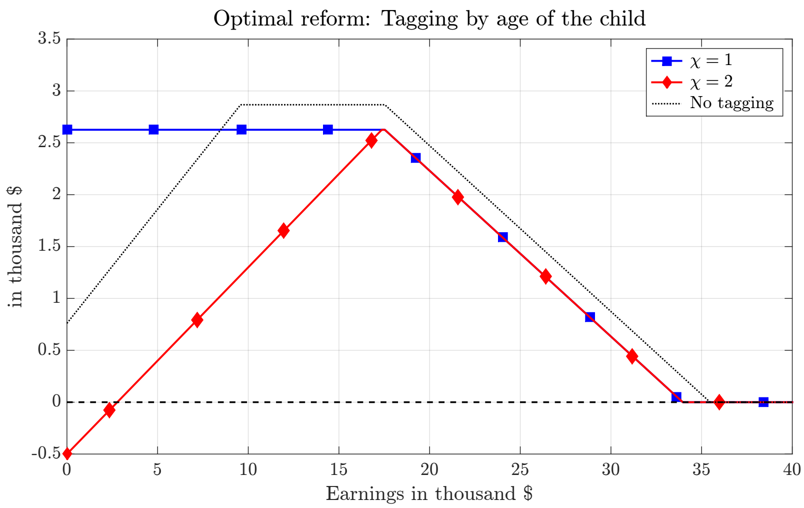
In this section we examine the potential gains from reforms that allow for tagging according to the age of the child (life stage). The merit of tagging, as pointed out in a seminal paper by Akerlof (1978), is that it makes the trade-off between guaranteed income and distortionary taxation more favorable. As is apparent from the optimal reform without tagging presented above, the government increases the guaranteed income at the cost of lowering work subsidies. Tagging eases this trade-off because it allows the government to increase the guaranteed income only to mothers in stage one while decreasing it to those in stage two, and to offer work subsidies only to the latter group of mothers. Since our model is dynamic, an income transfer policy that depends on the age of the child can thus ease the inefficiencies a mother in stage one is likely to face from lack of insurance, and at the same time provide her with incentives to work when she transits to stage two, when her demand for insurance is lower. This significantly improves the allocation of work and consumption across the two life stages.

Under tagging, the policy parameters that define the set of policy reforms are therefore indexed by χ, i.e, $(\kappa_0^\chi, \kappa_1^\chi, e_{I_1}^\chi)$ for $\chi = 1, 2$, which implies that we must find the values of these six parameters that maximize the ex-ante utility of entering single mothers. We let the government maximize over $(\kappa_0^\chi, \kappa_1^\chi)$ and adjust $e_{I_1}^\chi$ so that (i) revenue neutrality is attained, and (ii) the maximum tax credit is equal across the two groups of mothers. Note that feasible reforms allow for different levels of guaranteed income across the two groups, as well as for different profiles of working subsidies. The optimal policy is shown in column [3] of Table 5. κ_0^1 is equal to $2,626, which implies a guaranteed income of $8,950 for mothers in life stage one. For mothers in stage two κ_0^2 equals -$500, which amounts to a guaranteed income of $5,820 for these mothers, down from $6,320 under the

[8] Recall that this would be the intensive-margin Frisch elasticity of a single mother working the number of hours equal to the sample average and whose decision is not in a kink.

benchmark policy. The optimal earnings subsidy rate for working mothers in stage one, κ_1^1, is zero, which transforms the EITC schedule for these mothers into a TANF-like program (see Figure 5).

FIGURE 5– OPTIMAL REFORM WITH TAGGING BY AGE OF CHILD

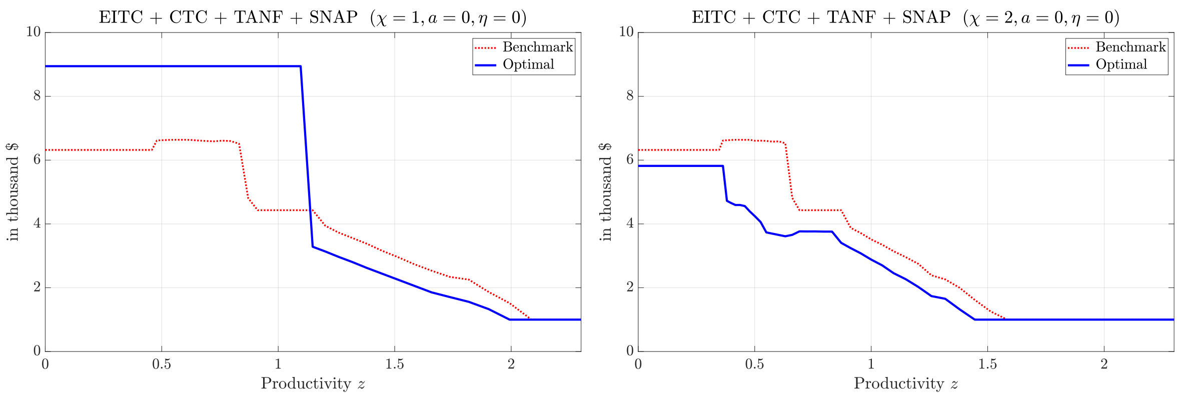


Working mothers in stage two get a subsidy rate of 18 percent up to earnings equal to $17,530. Interestingly, our restriction to reforms where the end of the phase-in region is bounded above by the benchmark value of e_{I_2} (which marks the beginning of the phase-out region) is binding. If allowed, the government would extend the phase-in region beyond $17,530 (see Figure 5). In sum, the optimal policy increases the guaranteed income and reduces work incentives to mothers in stage one. When they progress to stage two, the government reduces their guaranteed income and incentivizes them to work.

The difference in income transfers across life stages implied by the optimal reform is also shown in Figure 6. The left chart displays total income transfers from EITC, CTC, TANF and SNAP collected by a mother in stage one with no assets and no child care costs as a function of her labor productivity. The right chart displays total income transfers for a mother in stage two.

Summary statistics under the optimal policy are presented in column [4] of Table 6. The employment rate of mothers in stage one drops to 20.8 percent (due to the increased guaranteed income and the reduction in work incentives). The employment rate of mothers in stage two remains almost unchanged with respect to the level yielded by the benchmark policy. Conditional on working, average hours worked increase for both groups of mothers, relative to the benchmark. Likewise, conditional on working, average earnings also increase, especially for mothers in stage one. This follows from a composition effect: Under the optimal policy, only very productive mothers in stage one work. Total net transfers to mothers in stage one increase from $1,051 under the

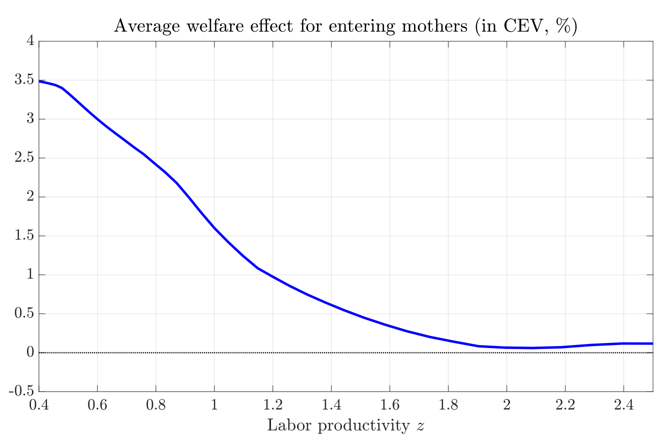
FIGURE 6– INCOME FROM EITC, CTC, TANF, SNAP (TAGGING BY CHILD AGE)



benchmark (on a per household basis) to $1,737 under the optimal policy. However, mothers in stage two cease to be net transfer recipients to become net tax payers under the optimal policy. Overall, net transfers to non-working mothers increase from $937 to $2,449; working mothers lose the $511 received in net transfers under the benchmark, to pay $1,002 in net taxes under the optimal policy. Disposable income becomes more equally distributed. The bottom quintile gets $9,755, compared to $8,102 under the benchmark policy.

We find substantial welfare gains from tagging. The welfare gain from this reform for entering mothers amounts to an increase by 1.73 percent in consumption in every period and state of nature. This is higher than the welfare gain from the optimal reform with no tagging, 0.63 percent. Furthermore, when we compute the welfare gain conditioning on the level of productivity drawn at entry, we find positive gains for all productivity levels (see Figure 7 below).

FIGURE 7– WELFARE EFFECTS FROM OPTIMAL REFORM (TAGGING)



5 Returns to Experience: Learning by Doing

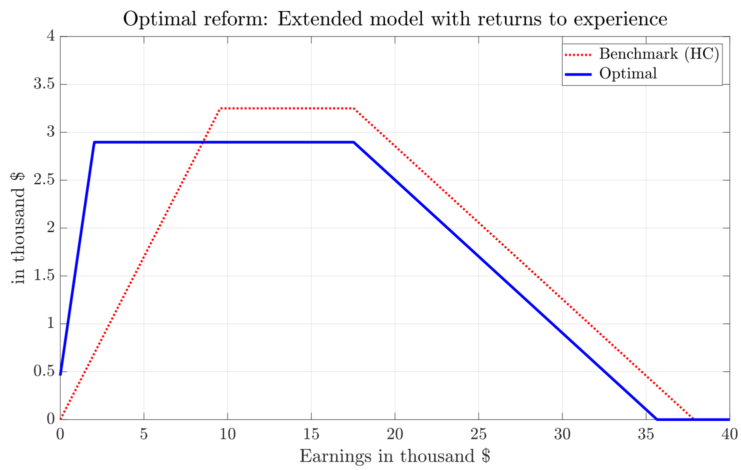
We extend the baseline model by introducing learning by doing. Namely, mothers may accumulate skills while working that increase their future average productivity. While evidence on the role of on-the-job learning for female labor supply is mixed (Olivetti 2006, Attanasio et al. 2008), our aim here is simply to assess whether the optimal transfer policy found in our baseline model would change if there are productivity gains from working, and if so how. The way we model learning-by-doing is rather crude—no human capital depreciation, and returns to experience are the only determinant of the increase in the average wage—and, hence, our quantitative results in this section are better interpreted as an upper bound. Of special interest to us is the robustness of our previous finding supporting a reform that increases the guaranteed income for single mothers.

Let us consider now two different levels of human capital, which are denoted by L (low) and H (high). Single mothers enter into their first stage of life with low human capital, and it is only by working that they can upgrade their human capital to level H. More specifically, we assume that mothers working full- and extra-time face a probability p of permanently increasing their human capital from L to H (no human capital depreciation).[9] With probability $1 - p$ they do not increase their human capital and start next period again with low human capital. The modeling of human capital accumulation as a stochastic process may be justified, for instance, by the different opportunities to learn offered across occupations. The value of parameter, p, is set equal to 0.20, implying an average of five years to attain the high level of human capital (H). The two wage rates, w^L and w^H, are set to match the average hourly earnings of mothers in stage one and two in our sample: \$12.01 and \$14.67, respectively. All other parameters are recalibrated to match the same set of moments as in the baseline model.

Before presenting our results for the optimal policy reform, it is useful to discuss the new trade-offs that emerge under learning by doing. As in the baseline model, increasing the guaranteed income allows mothers in stage one to further smooth marginal utility. But since now allocating time to market work can increase single mothers' human capital—which would make them more productive, and more likely to earn higher wages and remain employed in the future—a new force emerges: In order to incentivize single mothers in stage one to accumulate human capital, the government wants to reduce the guaranteed income and increase work subsidies. The way in which these conflicting forces are resolved in the optimal policy is shown in Figure 8. The value of κ_0 is \$460 (this parameter is equal to zero under the benchmark policy). That is, despite the potential productivity gains from working, it is still optimal to increase the guaranteed income by \$460 with

[9] We have experimented with an alternative assumption where part-time working mothers may increase their human capital as well and found our conclusions to be unaffected.

FIGURE 8–BENCHMARK AND OPTIMAL POLICY WITH LEARNING-BY-DOING

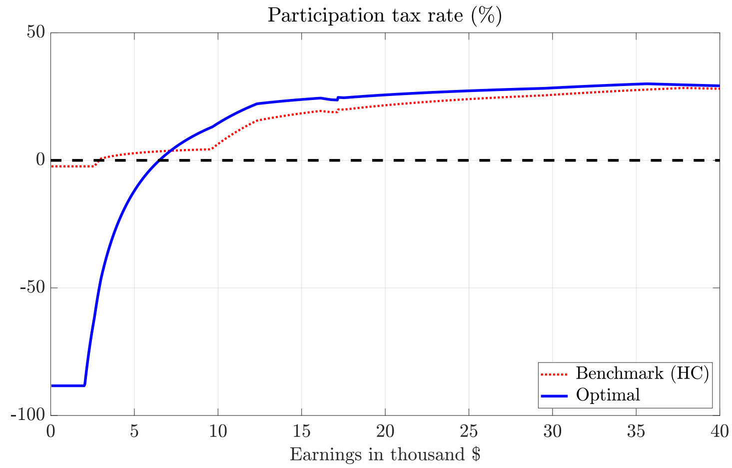


respect to the benchmark. (It should be noted that the optimal value of κ_0 is, however, lower than the one obtained in the model without learning by doing, \$760). The optimal earnings subsidy rate in the phase-in region, κ_1, is 120 percent, up from the 34 percent under the benchmark policy, and from the 22 percent under the optimal policy without learning by doing. However, the phase-in region is now much shorter: the value of e_{I_1} declines to \$2,414, from the \$9,559 in the benchmark policy. The high subsidy rate of the optimal policy at low earnings levels offsets the disincentives to work for medium-low productivity mothers introduced by the increased guaranteed income. This contributes to retaining more mothers in the labor market, especially among those in their first stage of life. In sum, the optimal policy combines a higher guaranteed income, so that wealth-poor mothers with very low productivity can smooth consumption, with higher incentives to work, so that medium-low productivity mothers join the labor force and get a chance of increasing human capital. Figure 9 below sheds light on the magnitude of these incentives. The consumption-equivalent welfare gain of adopting this policy for entering mothers is 0.6 percent.

The participation tax rate as a function of earnings under the optimal and benchmark policies are displayed in Figure 9. The figure shows large incentives to participate under the optimal policy (negative participation tax rates) for those earning up to \$7,000. For earnings levels above \$7,000, optimal participation tax rates are higher than those implied by the benchmark policy. Overall, the average participation tax rate in the steady-state solution under the optimal policy is 11 percent, compared to 17.8 percent of the optimal policy in the model without human capital accumulation.

Table 8 presents the effects of adopting the optimal reform on labor market variables, earnings, and net transfers as percentage changes from the values under the benchmark policy. For the sake of comparing these effects with those obtained in the baseline model without learning-by-doing, we also

FIGURE 9–PARTICIPATION TAX RATES



report those effects in the table. The optimal policy reduces employment rates, but much less than in the baseline model without learning-by-doing. For instance, mothers in life stage one reduce labor market participation by 2.57 percent, compared to 25.93 percent when there is no human capital accumulation. Average hours worked decline slightly, but average annual earnings increase. The change in the distribution of net transfers is qualitatively similar to that from adopting the optimal reform in the baseline model: net transfers decline for working mothers and increase for non-working mothers. Likewise, net transfers increase for mothers in life stage one and decline for mothers in life stage two. Average disposable income increases under the optimal policy. This is in contrast to the baseline model, where disposable income declines by about 5 percent with the optimal policy.

6 Concluding Remarks

This paper presents a normative analysis of the tax-transfer system to single mothers. Using the U.S. system of taxes and income transfers as our baseline, we develop a dynamic model of consumption/savings and labor supply to quantitatively characterize the optimal reform of this tax-transfer system. Our analytical framework is a standard model of heterogeneous agents, incomplete markets and borrowing constraints, extended to include fixed costs of working, child care cost risk, and child aging. We embed into this model two tax schemes (income and payroll taxes), two tax credits (the Earned Income Tax Credit and the Child Tax Credit), and two income assistance programs (the Temporary Assistance for Needy Families and the Supplemental Nutrition Assistance Program). Taxes, tax credits and assistance programs are modeled as they are in the U.S., including their kinks and non-convexities. By modeling an endogenous consumption-savings decision and

TABLE 8– PERCENTAGE CHANGE IN SUMMARY STATISTICS FROM BENCHMARK

Description	Sample [1]	% Change after Optimal Reform (Model with Learning-by-Doing) [2]	% Change after Optimal Reform (Model w/o Learning-by-Doing) [3]
Employment rate	All mothers	−5.57	−18.11
	$\chi = 1$	−2.57	−25.93
	$\chi = 2$	−6.67	−15.34
Part-time work	All mothers	−4.06	−34.51
Full-time work	All mothers	−0.27	6.55
Extra-time work	All mothers	15.06	28.16
Annual hours worked	All mothers	−1.60	6.44
	$\chi = 1$	−4.03	7.94
	$\chi = 2$	−0.46	5.66
Annual earnings	All mothers	2.89	13.80
	$\chi = 1$	0.13	18.70
	$\chi = 2$	4.09	11.66
Hourly earnings	All mothers	1.30	8.27
	$\chi = 1$	−0.16	10.40
	$\chi = 2$	2.06	7.21
Total net transfer	All mothers	−−	−−
	Working	−58.17	−208.61
	Non working	41.27	113.87
	$\chi = 1$	0.11	1.33
	$\chi = 2$	−0.19	−3.76
Disposable income	All mothers	0.33	−5.15
Quintile 1		0.92	3.41
Quintile 2		0.65	22.99
Quintile 3		−0.84	−3.87
Quintile 4		0.17	−1.48
Quintile 5		0.12	0.48
Wealth	All mothers	−0.90	−3.99
Frac. with no assets	All mothers	−6.15	4.58
Frac. with wealth $\leq \$2,000$	All mothers	0.00	3.07
Corr(hours,productivity)	All mothers	13.38	8.39

NOTES: †See subsection 4.4.

uninsurable earnings and child care risks, the optimal tax policy problem in our model also assesses the differing needs for insurance of single mothers depending on the age of their child. (Single mothers of a pre-schooling child are more likely to be credit constrained and hence are willing to trade off less out-of-work income support and higher work subsidies in the future for more out-of-work support and lower subsidies today).

We find that the optimal out-of-work income support is higher than the current one. When policy cannot be made dependent on the age of the child, the optimal reform prescribes an increase in out-of-work income support and a reduction in the subsidy rate to low-wage earners. When policy can depend on the age of the child, the optimal reform provides more insurance to single mothers of a pre-schooling child by increasing out-of-work income support but eliminates work subsidies for them. For single mothers of a school-age child, the optimal reform prescribes lower out-of-work income support but positive working subsidies for low-wage earners.

Our results highlight the important role of savings for the optimal taxation of single-parent households. While our structural model contains the standard behavioral margins considered in the literature on optimal taxation (consumption/savings and the extensive and intensive margins of labor supply), we have left for future research margins that might be of interest for the case of single mothers. For instance, in our model family composition is kept fixed, as we model neither fertility nor family formation decisions. Regarding the effects of taxes and transfers on fertility, the empirical literature has found no or little effects (Baughman and Dickert-Conlin 2009, Crump et al. 2011). Regarding family formation, while taxes and transfers have been found to affect the marriage rate of low-income individuals (Herbst 2011), evidence on the effects on divorce is mixed. Whether endogenous fertility and family formation have quantitatively important implications for the optimal taxation of single mothers is an important question for future research.

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Appendix A

Taxes, Tax Credits, and Income Assistance Programs

In this appendix we describe the U.S. federal individual income tax scheme, the payroll tax, two tax credits and two income transfer programs to assist low-income households. We embed these tax and transfer schemes into our model as they are described here, including all the discontinuities and kinks.

INCOME AND PAYROLL TAXES

Single mothers in our model file with the Internal Revenue Service (IRS) using two filling statuses, j: head of household, ($\hbar$), for single mothers in stages one and two with one dependent; and single, (s), for single mothers in stage three, as their child is no longer eligible as a dependent. The filing status affects both taxes paid (tax rates and deductions), as well as eligibility and benefits for tax credits.

A single mother's income is made up of earnings, e, and capital income, ra, where r is the return on investment and a is the level of assets. Income taxes before credits owed by a tax filer under filing status $j = \hbar, s$, with income $y = e + ra$ are given by

$$T^j(y) \;=\; \sum_{i=1}^{7} \tau_y^{j,i} \max\{\min\{y - d_T^j - \xi_T^j, b^{j,i}\} - b^{j,i-1}, 0\},$$

where $b^{j,i} \geq 0$ are parameters characterizing the seven income brackets in the federal individual income tax code, and $\tau_y^{j,i}$ are the corresponding tax rates. The upper bound for the last bracket, $b^{j,7}$, is set to a very large value such that taxable income for any household is below this limit. The remaining values, $b^{j,i}$ for i from 1 to 6, are the break points between the different income brackets. The income tax deduction is denoted by d_T^j and personal exemptions by ξ_T^j.

Payroll taxes are denoted by $T_p(e) = \tau_p \min\{e, \bar{e}\}$, where $\tau_p = \tau_{p,\mathrm{SS}} + \tau_{p,\mathrm{ME}}$ is the employee's tax rate (the sum of social security and medicare tax rates), and $\bar{e}$ is the payroll tax cap.

TAX CREDITS

The Earned Income Tax Credit (EITC). The Earned Income Tax Credit is a refundable credit. Eligibility is determined by the following conditions: (i) Investment income, ra, cannot exceed a level, say $\bar{\mathrm{ra}}_I$; (ii) Income (earned plus non-earned income) cannot exceed a level, say y_I^j, which depends on the filing status (note that in our model all mothers filling with a head of household status have one dependent, and, hence, we do not need additional notation for the number of dependents). The EITC-eligibility set of a tax filer under filing status $j = \hbar, s$ is

$$EES \equiv \{ra \leq \bar{\mathrm{ra}}_I\} \cap \{e + ra \leq \mathrm{y}_I^j\}. \tag{6.1}$$

The amount of the credit accruing to a tax filer with assets a, earned income e, and filing under status j is given by

$$EITC^j(a,e) = \begin{cases} \kappa_1^j e & \text{if} \quad 0 \leq e \leq \mathrm{e}_{I_1}^j \text{ and } a \in EES \\ \kappa_1^j \mathrm{e}_{I_1}^j & \text{if} \quad \mathrm{e}_{I_1}^j \leq e \leq \mathrm{e}_{I_2}^j \text{ and } a \in EES \\ \max\{\kappa_1^j \mathrm{e}_{I_1}^j - \kappa_2^j(e - \mathrm{e}_{I_2}^j), 0\} & \text{if} \quad e \geq \mathrm{e}_{I_2}^j \text{ and } a \in EES, \end{cases}$$

and EITC(a,e) $= 0$ if $(a, e) \notin EES$. Parameters κ_1^j are the earnings subsidy rates in the phase-in region, and κ_2^j are the phase-out rates. The thresholds, $\mathrm{e}_{I_1}^j$ and $\mathrm{e}_{I_2}^j$, mark the end of the phase-in region and the beginning of the phase-out region, respectively. In the region between these two thresholds, the credit is constant at its maximum value $\kappa_1^j \mathrm{e}_{I_1}^j$. Note that both the credit rates and the earnings thresholds depend on the number of qualifying children and the filing status. However, the maximum level of investment income for program eligibility, $\bar{\mathrm{ra}}_I$, does not depend on either of the two. (Figure A1 below displays the EITC schedule for the 2013 tax returns.)

The Child Tax Credit (CTC). The non-refundable component of the child tax credit for a tax filer under status $j = \hbar$ and income y is

$$CTC^{\hbar}(y) = \begin{cases} \theta & \text{if} \qquad y \leq \mathrm{y}_{CTC}^{\hbar} \\ \max\{\theta - \varrho(y - \mathrm{y}_{CTC}^{\hbar}),\ 0\} & \text{if} \qquad y > \mathrm{y}_{CTC}^{\hbar}, \end{cases}$$

where θ is the subsidy per child and $\mathrm{y}_{CTC}^{\hbar}$ is the income level at which the child tax credit starts being phased out. Parameter ϱ characterizes the child tax credit phase-out rate.

If the non-refundable component of the child tax credit, $CTC^{\hbar}(y)$, is lower than the tax liability, $T^{\hbar}(y)$, then this liability is reduced by the amount of the child tax credit. If the child tax credit is higher than the liability, then the liability is reduced to zero and the filer can apply for the refundable component of the Child Tax Credit, i.e. the Additional Child Tax Credit (ACTC). The additional child tax credit for a mother with one eligible child is

$$ACTC^{\hbar}(y, e) = \min\left\{CTC^{\hbar}(y) - T^{\hbar}(y),\ \max\{\phi(e - \delta), 0\}\right\} \tag{6.2}$$

where ϕ and δ are parameters. (Figure A1 below displays the CTC schedule for the 2013 tax returns.) With some abuse of notation, we refer to the sum of the two components (the non-refundable and the refundable) as the Child Tax Credit and write this sum as $CTC^j(a, e)$.

INCOME ASSISTANCE PROGRAMS

Temporary Assistance for Needy Families (TANF). This is a program to assist families with dependent children, and, hence, only mothers in stages one and two of our model may be eligible. Despite variation across states, many features of the program are common across most states. Eligibility and benefits are determined by categorical and quantitative variables of the assistance unit on a monthly basis. For the sake of our analysis, we consider assistance units made up of a single mother with one dependent child. Financial eligibility requirements include: (i) Assets cannot exceed a certain limit, say a_B.[10] (ii) Gross family income cannot exceed y_{B_1}, say. Gross income includes earned and non-earned income, such as interests and child support income. (iii) Net family income cannot exceed y_{B_2}. Net income for the purpose of determining TANF eligibility is computed as

$$\iota_B(a,e,h,\eta) = \left(e - d_{B_1}\mathbb{1}_{\{h>0\}} - d_{B_2}\Gamma(h,\eta) - d_{B_3}\right)\sigma_B + ra + \vartheta, \tag{6.2}$$

where $\sigma_B < 1$ is a parameter that introduces an earned income disregard; d_{B_1} is a work deduction, $\mathbb{1}_{\{h>0\}}$ is an indicator function which takes value 1 if hours worked are strictly positive; d_{B_2} is a child care deduction, which is set as a fraction of child care costs incurred while working, Γ; and d_{B_3} is a fixed deduction. Parameter ϑ is child support.

These three financial requirements define the TANF-eligibility set of a single mother with one dependent child as

$$TES \equiv \{a \leq \mathrm{a}_B\} \cap \{e + ra + \vartheta \leq \mathrm{y}_{B_1}\} \cap \{\iota_B(a,e,h,\eta) \leq \mathrm{y}_{B_2}\}. \tag{6.2}$$

If eligible, the income transfer is determined by a standard of need and net family income, with a maximum payment set by a payment standard. That is, an eligible single mother with one dependent child is entitled to TANF benefits

$$TANF(a,e,h,\eta) = \min\left\{\bar{B},\ \max\{[S - \iota_B(a,e,h,\eta)] \times \varsigma, 0\}\right\}, \tag{6.2}$$

where $\bar{B}$ is the maximum transfer; S is the standard of need for that family, which is set as a percentage of the federal poverty level; $\iota_B(a,e,h,\eta)$ is net income as defined above; and ς is a parameter that controls when, and the rate at which, transfers are phased out. (Figure A1 below displays the 2013 TANF schedule.)

[10] Eight states have eliminated TANF asset limits (Ohio, Louisiana, Colorado, Hawaii, Illinois, Virginia, Alabama and Maryland). Other states do not impose limits on certain assets, such as retirement and education accounts and vehicles.

TANF has work requirements and time limits, typically of 60 months, to receive TANF benefits. However, the extent of enforceability of these limits varies widely across states. Besides a number of exemptions from time limits, states are allowed to extend assistance beyond these limits to up to 20% of their caseload.

Supplemental Nutrition Assistance Program (SNAP). While this is a federal in-kind transfer program, we follow many studies in considering the food coupons near-cash transfers. For SNAP, an assistance unit is an individual or a group of individuals who live together and purchase and prepare meals together. In our model there are two distinct types of assistance units: single mothers in stages 1 and 2, which are made up of two individuals; and single mothers in stage 3, which are made up of one individual. For an assistance unit with $\jmath$ individuals ($\jmath = 1, 2$), eligibility is determined by (i) a resource limit, a_F; (ii) a gross income limit, $\mathrm{y}^{\jmath}_{F_1}$, where gross income is defined to include earned and non-earned income, such as investment income, child support and income received from TANF; and (iii) a net income limit, $\mathrm{y}^{\jmath}_{F_2}$. Net income is computed as gross income minus an earned income disregard, a child care deduction when needed for work, d_{F_1}, and a standard deduction

$$\iota_F(a, e, h, \eta) = e \cdot \sigma_F + ra + \vartheta + B(a, e) - d_{F_1}\Gamma(h, \eta) - d_{F_2}, \tag{6.2}$$

where $1 - \sigma_F$ is the earned income disregard.

In sum, the SNAP-eligibility set of an assistance unit of type $\jmath = 1, 2$ is

$$SES \equiv \{a \leq \mathrm{a}_F\} \cap \{e + ra + \vartheta + B(a, e) \leq \mathrm{y}^{\jmath}_{F_1}\} \cap \{\iota_F(a, e, h, \eta) \leq \mathrm{y}^{\jmath}_{F_2}\}. \tag{6.2}$$

If a single mother receives TANF income she does not need to pass the income tests, and is immediately entitled to SNAP transfers provided she meets the resource test.

SNAP benefits are calculated by subtracting the household's expected contribution towards food, i.e. υ times net income, from a maximum allotment for the household. That is, an eligible assistance unit of type $\jmath$ is entitled to SNAP benefits

$$SNAP^{\jmath}(a, e, h, \eta) = \max\left\{\bar{F}^{\jmath} - \upsilon \cdot \iota_F(a, e, h, \eta), \underline{F}^{\jmath}\right\}, \tag{6.2}$$

where $\bar{F}^{\jmath}$ is the maximum allotment a $\jmath$-type assistance unit can receive from SNAP, and $\underline{F}^{\jmath}$ is the minimum benefit an eligible $\jmath$-type unit.

In Figure A1 we display the EITC, CTC, TANF and SNAP for an eligible single mother with one child as a function of her annual earnings. The figure is meant to illustrate the kinks generated by these programs in the budget constraints of single mothers.

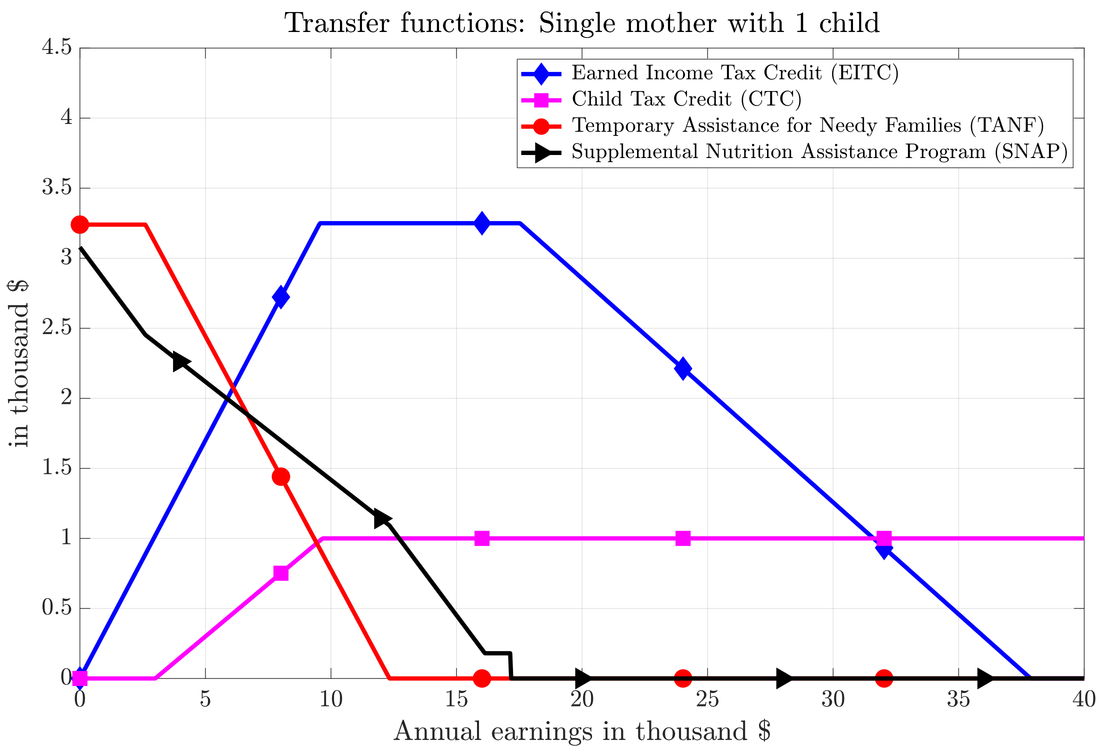

FIGURE A1. *Notes:* Income tranfers from EITC, CTC, TANF and SNAP collected by a single mother of one child with no assets and no child care costs as a function of annual earnings .

Appendix B

Tax-transfer parameter values

This Appendix presents the parameter values of the 2013 federal income tax schedule, payroll taxes, and the four transfer programs in our model (the Earned Income Tax Credit, the Child Tax Credit, the Temporary Assistance for Needy Families and the Supplemental Nutrition Assistance Program).

Income and Payroll Taxes

Table B1 presents the income tax brackets for tax filers under the *single* status and for tax filers under the *head of household* status.

TABLE B1—INCOME BRACKETS (ALL VALUES IN $)

Bracket	Parameter	Single ($j = s$)	Head of household ($j = \hbar$)
1	$b^{j,0}$	0	0
2	$b^{j,1}$	8,925	12,750
3	$b^{j,2}$	36,250	48,600
4	$b^{j,3}$	87,850	125,450
5	$b^{j,4}$	183,250	203,150
6	$b^{j,5}$	398,350	398,350
7	$b^{j,6}$	400,000	425,000

Source: 2013 income brackets for federal income taxes, from IRS website.

Table B2 presents the standard deduction, the personal exemption, and the marginal tax rates in the seven income tax brackets. The table also presents the payroll tax rates and the tax cap.

TABLE B2—INCOME AND PAYROLL TAX RATES

Description	Comment	Parameter	Value
Standard deduction (in \$)	Single	d_T^s	6,100
Standard deduction (in \$)	Head of household	$d_T^{\hbar}$	8,950
Personal exemption (in \$)	Per person	ξ_T	3,900
Marginal tax rate	Bracket 1	τ_y^1	0.10
Marginal tax rate	Bracket 2	τ_y^2	0.15
Marginal tax rate	Bracket 3	τ_y^3	0.25
Marginal tax rate	Bracket 4	τ_y^4	0.28
Marginal tax rate	Bracket 5	τ_y^5	0.33
Marginal tax rate	Bracket 6	τ_y^6	0.35
Marginal tax rate	Bracket 7	τ_y^7	0.396
Social Security tax	Employee's share	$\tau_{p,SS}$	0.0620
Medicare tax	Employee's share	$\tau_{p,MA}$	0.0145
Social Security cap (in \$)	Earnings cap	$\bar{e}$	113,700

Source: 2013 standard deductions, federal income tax rates and payroll taxes, from IRS website.

The Earned Income Tax Credit (EITC)

EITC eligibility is determined by wealth and income. Table B3 below presents the eligibility thresholds for single mothers with one dependent (filing as *head of household*) and for single mothers without dependents (filing as *single*).

TABLE B3—EARNED INCOME TAX CREDIT: ELIGIBILITY

	Max. investment income, $\mathrm{r\bar{a}}_I$ (\$)	Max. total income, y_I (\$)
Mothers in stages 1 and 2 (one child, $j = \hbar$)	$3,300$	$37,870$
Mothers in stage 3 (no child, $j = s$)	$3,300$	$14,340$

Source: Investment and total income limits for 2013 EITC eligibility, from IRS website.

The amount of the credit is determined by the level of earnings and the number of dependents. Table B4 presents the credit rates and the earning threshold that determine the three EITC regions (phase-in, plateau, and phase-out).

TABLE B4—EARNED INCOME TAX CREDIT: CREDIT RATES AND EARNINGS THRESHOLDS

	Phase-in rate, κ_1 (%)	Earnings end phase-in, e_{I_1}($)	Earnings beginning phase-out, e_{I_2}($)	Phase-out rate, κ_2 (%)
Mothers in stages 1 and 2 (one child, $j = \hbar$)	34.0	9,550	17,550	15.9
Mothers in stage 3 (no child, $j = s$)	7.65	6,350	8,000	7.65

Source: Subsidy rates and earnings thresholds for 2013 EITC, from IRS website.

The Child Tax Credit and the Additional Child Tax Credit

In our model, only single mothers with one dependent child (stages 1 and 2) are eligible for the (non-refundable) CTC and the (refundable) ACTC. Table 5 presents the parameters determining eligibility and the amount of the credit.

TABLE B5—CHILD TAX CREDIT: CREDIT RATES & INCOME AND EARNINGS THRESHOLDS

Description	Parameter	Value
Credit per child	θ	1,000
Phase-out income threshold	y_{CTC}	75,000
Phase-out rate	ϱ	5%
Earnings limit (ACTC)	δ	3,000
Weight on earnings gap (ACTC)	ϕ	0.15

Source: Credit rates and income thresholds for 2013 CTC and ACTC, from IRS website.

Temporary Assistance for Needy Families (TANF)

In our model, only single mothers with one dependent child (stages 1 and 2) are eligible for TANF. Table B6 presents the parameters determining eligibility and benefits from TANF.

TABLE B6—TEMPORARY ASSISTANCE FOR NEEDY FAMILIES (TANF)

Description	Parameter	Size assistance unit	
		1 person	2 persons
Standard of need	S	638	855
Work deduction (per worker)	d_{B1}	90	90
Child care deduction	d_{B2}	0.5	0.5
General deduction	d_{B3}	30	30
Maximum grant	$\bar{B}$	201	270
Gross income test	y_{B1}	1180	1581
Net income test	y_{B2}	638	855
Asset test	a_B	2,000	2,000
Generosity	ς	0.5	0.5
Earned income disregard	σ_B	2/3	2/3

Source: Income and asset limits, deductions and benefits for 2013 TANF, from the state of Delaware's website. TANF is a monthly program and the dollar amounts in this table are monthly values. Since the length of a period in our model is one year, we annualize these values to fit our model.

Supplemental Nutrition Assistance Program (SNAP)

Single mothers in stages 1, 2 and 3 can apply to SNAP. Table B7 presents the parameters determining eligibility and benefits from SNAP.

TABLE B7—SUPPLEMENTAL NUTRITION ASSISTANCE PROGRAM (SNAP)

Description	Parameter	Size of assistance unit	
		1 person	2 persons
Asset test	a_F	2,000	2,000
Gross income test	y^J_{F1}	1,245	1,681
Net income test	y^J_{F2}	958	1,293
Child care deduction	d_{F1}	–	0.5
Standard deduction	d_{F2}	152	152
Earned income disregard	σ_F	0.8	0.8
Maximum allotment	$\bar{F}^J$	200	367
Weight on net income	υ	0.3	0.3
Minimum benefit	$\underline{F}^J$	15	15

Source: Income and asset limits, deductions and benefits for 2013 SNAP, from the U.S. Department of Agriculture, Food and Nutrition Service's website. SNAP is a monthly program and the dollar amounts in this table are monthly values. Since the length of a period in our model is one year, we annualize these values to fit our model.

Appendix C

In this appendix we provide a detailed description of the welfare decomposition yielding the numbers reported in Table 5 in the main text. We partially follow Conesa, Kitao and Krueger (2009) who describe a similar procedure in footnote 18. It should be noted, however, that their Cobb-Douglas specification for the utility function allows them to compute consumption equivalent variations in closed form. By contrast, with our additively separable utility function, this is not possible and we must solve nonlinear equations numerically. Let (c_0, h_0) denote the consumption-labor allocation in the benchmark economy, and let (c_*, h_*) denote the consumption-labor allocation under the optimal reform. Then the total welfare gain from implementing the optimal reform, measured in terms of a consumption equivalent variation CEV, is defined as

$$W(c_*, h_*) \ = \ W(c_0(1 + CEV), h_0).$$

The solution to this equation, CEV, is the aggregate welfare gain for an entering mother under the optimal reform. We proceed by decomposing this welfare gain into a component stemming from the change in consumption from c_0 to c_*, and another component stemming from the change in hours worked from h_0 to h_*. These two components are denoted by CEV_C and CEV_H, respectively, and they are defined as:

$$\begin{aligned} W(c_*, h_0) \ &= \ W(c_0(1 + CEV_C), h_0) \\ W(c_*, h_*) \ &= \ W(c_*(1 + CEV_H), h_0). \end{aligned}$$

Note that, in contrast to Conesa, Kitao and Krueger (2009), with our additively separable utility function, the relation $1 + CEV = (1 + CEV_C)(1 + CEV_H)$ generally does not hold. We then decompose the welfare effect stemming from the change in consumption, CEV_C, into a level effect CEV_{CL} and a distribution effect CEV_{CD}, defined as follows:

$$\begin{aligned} W(\hat{c}_0, h_0) \ &= \ W(c_0(1 + CEV_{CL}), h_0) \\ W(c_*, h_0) \ &= \ W(\hat{c}_0(1 + CEV_{CD}), h_0), \end{aligned}$$

where $\hat{c}_0 = (C_*/C_0)c_0$ is the consumption allocation resulting from scaling the allocation c_0 by the change in aggregate consumption. Similarly, the welfare effect stemming from the change in hours worked, CEV_H, can be decomposed into CEV_{HL} and CEV_{HD}:

$$\begin{aligned} W(c_*, \hat{h}_0) \ &= \ W(c_*(1 + CEV_{HL}), h_0) \\ W(c_*, h_*) \ &= \ W(c_*(1 + CEV_{HD}), \hat{h}_0), \end{aligned}$$

where $\hat{h}_0 = (H_*/H_0)h_0$ is the non-market time allocation resulting from scaling the allocation h_0 by the change in aggregate hours worked.