On the Optimality of U.S. Fiscal Policy: 1960-2010

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Abstract

This paper assesses the optimality of U.S. fiscal policy from 1960 to 2010. With this purpose, we make use of a tractable neoclassical economy to obtain the optimal, time-consistent fiscal policy set by a benevolent government. We then compare the model's prescriptions for income tax rates and government consumption with their empirical counterparts observed in the U.S. We find that from 1960 to 2000 U.S. income taxation and government consumption evolved according to the model's prescriptions. However, starting in the early 2000s and for the rest of the decade, U.S. fiscal policy trended in a direction opposite to the policy that would have been set by a benevolent government. Specifically, between 2001 and 2010 the U.S. income tax rate went down by 19.5 percent, while our model prescribes an increase of 8 percent in the same period. U.S. government consumption as a share of GDP went up by 19.3 percent, while our model prescribes a decrease of 6.4 percent in this period.

JEL Codes: E62, H24, H40, H50

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1 Introduction

Measures of U.S. income tax rates and government consumption expenditure show significant variation in these two variables over the period from 1960 to 2010. In particular, synthetic income tax rates (a combination of personal current taxation, corporate income taxation and contributions to government social insurance) exhibit two markedly different trends during this period. First, between 1960 and 2000, income tax rates display an upward trend, from 17 percent in the early 1960s to 23 percent in the late 1990s. Then, from the early 2000s to 2010, income tax rates decline for most of the decade, reversing the previous increases. On the other hand, the evolution of government consumption as a share of adjusted GDP shows: (i) An increase during the decade of the 1960s; (ii) a downward trend from the early 1970s to the late 1990s, which reduced the government consumption-to-GDP ratio from 19.5 percent in 1970 to 15 percent in 2000; and finally (iii) a sharp increase in the 2000s, to reach a ratio of 18.5 percent in 2010.

In this paper, we assess whether the trends in income tax rates and government consumption reported above were optimal, conditional on household consumption in this period. Assessing optimality requires a normative framework, and, hence, the adoption of a number of assumptions including the level of aggregation and the restrictions faced by the government when setting fiscal policy variables. Our analysis assumes a neoclassical, two-sector model of consumption and savings with a representative household and identical firms. In modeling the government, we introduce a novel restriction that, despite its prevalence, has been largely overlooked in the economic literature. Namely, we assume that the government lacks both intra- and inter-period commitment to fiscal policy. This implies that the government can renege from the policy already announced for the current period, and change taxes retroactively. In the U.S., retroactive tax reforms are constitutional. In several rulings, the Supreme Court of the United States has rejected constitutional challenges to federal and state retroactive tax legislation. (As recently as May 22, 2017, the U.S. Supreme Court declined to hear a challenge based on due process to a Michigan tax law that retroactively changes businesses' taxation.)[1]

To help us devise a strategy to meaningfully assess the optimality of U.S. tax rates and government consumption, we draw on results from Ortigueira and Pereira (2017). Specifically, these authors show that when the government is allowed to change taxes retroactively, expectations may play a role in policy making. Indeed, they find, using a standard neoclassical economy, a multiplicity of expectations-driven, Markov-perfect equilibria, Hence, economic fundamentals do

[1]For a detailed account of the major retroactive tax reforms by the U.S. federal government, see Romer and Romer 2009.

not uniquely pin down consumption, savings, and fiscal policy variables. The indeterminacy is resolved by households' expectations on fiscal policy. In the current paper, we note that from the set of Markov-perfect equilibria arising in economies where retroactive tax reforms are constitutional a unique relationship between household consumption and optimal fiscal policy can be established. Hence, we take the path of U.S. household consumption as a primitive, and then obtain the optimal fiscal policy associated with that consumption path. Put differently, U.S. household consumption is used to select one equilibrium from the model's set of Markov-prefect equilibria. Fiscal policy along the thus selected equilibrium is then compared to the observed fiscal policy. It must therefore be stressed that our assessment of the U.S. fiscal policy between 1960 and 2010 is conditional on the path of household consumption in this period.

The analytical framework used in this paper is a standard neoclassical, two-sector model of capital accumulation. Identical, infinitely-lived households make consumption/savigns decisions, supply labor to firms and pay direct and indirect taxes to the government. Firms produce a homogeneous good that can be either consumed or used as an input in the production of physical capital. We assume a convex adjustment technology for converting the homogeneous good into the capital good, as in Lucas and Prescott (1971). In addition to rendering the price of capital endogenous, this convex technology allows us to obtain the Markov-perfect equilibria of the model in closed form. The benevolent government chooses fiscal policy, consisting of the level of expenditure on a public good, income taxation and debt issues. For simplicity, we assume that consumption and investment taxes, and transfers to households are exogenous. Because the government lacks commitment to policy, fiscal policy decisions are made sequentially. Furthermore, since the government can retroactively change its policy for the current period any moment within the fiscal year, households cannot condition their consumption/savings decisions on the pre-announced policy.[2]

To construct the empirical counterparts of the fiscal policy variables in our theory, we use annual data from the National Income and Product Accounts (NIPA) of the Bureau of Economic Analysis. We construct tax rates on consumption and investment, income tax rates and the government consumption-to-GDP ratio from 1960 to 2010. When these variables are compared with their respective optimal values generated from the model, we obtain the following findings. During the forty-year period between 1960 and 2000, actual and optimal policies are fairly close to each other. When actual and optimal income tax rates are compared, both trend upward

[2]A previous strand of the literature has focused on the implications of the government's ability to carry out *prospective* changes to fiscal policy, but assuming that *retroactive* changes are disallowed. Early contributions to this literature are Ortigueira (2006), Klein, Krusell and Ríos-Rull (2008) and Martin (2009).

during this period. Although for a few years in the 1970s and the 1990s there is a gap of almost four percentage points between the two rates, for most other years, especially in the 1980s, the two series are very close to each other. Similarly, actual and optimal government consumption-to-GDP ratios move closely together during this forty-year period. The two ratios increase during the 1960s and then decrease from 1970 to 2000. However, we get strikingly different results for the ten-year period between 2001 and 2010. Actual income tax rates begin to decline in the early 2000s, whereas optimal rates continue to display an upward trend, thus creating an increasing gap between actual and optimal tax rates. Actual and optimal government consumption-to-GDP ratios also follow opposite trends during the decade of the 2000s. Actual ratios shoot up in the early 2000s and continue to increase during the decade. In contrast, optimal ratios continue to display a downward trend, giving rise to an increasing gap between the two. In sum, when our model is evaluated at the values of household consumption observed in the U.S., it offers a clear assessment of the optimality of U.S. income tax rates and government consumption-to-GDP ratios. From 1960 to 2000 U.S. fiscal policy is close to that prescribed by the model, and it can accordingly be considered optimal. However, from 2001 to 2010 U.S. fiscal policy departs from the model's prescriptions and it is hence deemed non optimal.

The remainder of the paper is organized as follows. Section 2 outlines our model economy, presents the maximization problems solved by the households and by the government, and characterizes Markov-perfect equilibria. In Section 3 we construct tax rates on consumption, tax rates on investment, income tax rates and the government consumption-to-GDP ratios of the U.S. economy from 1960 to 2010. In Section 4 we calibrate our model, generate optimal fiscal policy in Markov-perfect equilibrium and then compare this optimal policy with U.S. policy. Section 5 presents our concluding remarks. An Appendix contains the proofs.

2 A Simple Model of Optimal Fiscal Policy

We present a simple model of optimal fiscal policy under no commitment. Our framework is a two-sector production model of capital accumulation with a representative household. A benevolent government provides a valued public good and makes transfers to households. In order to finance the provision of the public good and the transfers, the government levies taxes and issues public debt. There are three taxes available: a consumption tax, an investment tax, and an income tax. We set the level of transfers and the tax rates on consumption and investment exogenously and let the government choose the level of expenditure on the public good, G_t, the tax rate on income, τ_t, and debt issues, B_{t+1}, which mature in period $t+1$.

We begin by describing the objective and the restrictions faced by each agent in this economy. We then characterize optimal fiscal policy in Markov-perfect equilibrium.

2.1 Production

There are two sectors of production. One sector produces an homogeneous good which is consumed by the households as a private good, C_t, used as an input in the sector that produces the capital good, X_t, and consumed by the households as a government-provided public good, G_t. Production of this good is described by the Cobb-Douglas production function

$$C_t + X_t + G_t = AK_t^{\alpha}. \tag{1}$$

The other sector produces the capital good. Production of the capital good is described by Cobb-Douglas production function

$$K_{t+1} = DX_t^{\lambda} K_t^{1-\lambda}, \tag{2}$$

where $D > 0$ and $0 \leq \lambda \leq 1$ are parameters.[3] The capital production technology (2) nests three different specifications depending on the values of D and λ: *(i)* A fixed capital stock, $D = 1$ and $\lambda = 0$; *(ii)* full capital depreciation, $\lambda = 1$; and *(iii)* partial capital depreciation with investment adjustment costs, $0 < \lambda < 1$. We will focus on this latter specification, where the decreasing returns to investment can be viewed as stemming from adjustment costs. Indeed, in this case the capital production technology in (2) is equivalent, up to a second-order approximation around the steady-state, to the commonly-used law of motion for capital under quadratic adjustment costs

$$K_{t+1} = (1-\delta)K_t + X_t - \frac{\chi}{2}\left(\frac{X_t}{K_t} - \delta\right)^2 K_t, \tag{3}$$

when

$$\delta = \lambda, \;\; \chi = (1-\lambda)/\lambda \text{ and } D = \lambda^{-\lambda}. \tag{4}$$

This two-sector representation of production, and the exposition in this section, follows closely that of Abel (2003). The price of one unit of the capital good in terms of the homogeneous good,

[3] This log-linear technology is a particular case of the law of motion for physical capital assumed by Lucas and Prescott (1971). It has been used by Hercowitz and Sampson (1991) to study business cycles in a model of endogenous growth. Abel (2003) embeds this technology into an overlapping generations model to assess the effects of a baby boom on stock prices and capital accumulation. More recently, Jin (2012) adopts this technology in his analysis of trade and international capital flows.

p_t, is given by

$$p_t = \left(\lambda D X_t^{\lambda-1} K_t^{1-\lambda}\right)^{-1}, \tag{5}$$

which is the increase in X_t needed to produce one more unit of K_{t+1}.

The demand of physical capital is given by

$$r_t = \alpha A K_t^{\alpha-1} + p_t(1-\lambda) D X_t^{\lambda} K_t^{-\lambda}, \tag{6}$$

where r_t is the rental price of physical capital in units of the homogeneous good. From the zero-profit condition we get the wage as

$$\omega_t = (1-\alpha) A K_t^{\alpha}. \tag{7}$$

While our use of production technology (2) is motivated by analytical tractability, we will exploit its equivalence to (3) to calibrate our model economy.

2.2 Households

There is a continuum of identical households with measure one. Each household chooses consumption and savings in order to maximize lifetime utility

$$\sum_{t=0}^{\infty} \beta^t (\ln c_t + \theta \ln G_t) \quad \text{with } \theta > 0, \tag{8}$$

where c_t denotes household consumption of the private good, and G_t is a government-provided public good.

Households' asset holdings are made up of physical capital, k_t, which is rented to firms at the rate r_t, and government bonds, b_t. The period budget constraint faced by a household is

$$(1+\tau_{c,t})c_t + (1+\tau_{x,t})p_t k_{t+1} + q_t b_{t+1} = b_t + (1-\tau_t)[w_t + r_t k_t] + T_t, \tag{9}$$

where $\tau_{c,t}$ and $\tau_{x,t}$ are the tax rates on consumption and investment, respectively. q_t is the price of government bonds issued in period t. T_t denotes the transfers received from the government. Household income is taxed at the rate τ_t. We assume that neither income earned on public debt nor transfers are subject to taxation.

2.3 Government

The fiscal authority chooses the level of expenditure on the public good and its financing through income taxes and debt. As indicated above, transfers to households, and tax rates on consumption and investment are assumed to be exogenously determined, and therefore are not part of the fiscal authority's decision problem.

The fiscal authority is benevolent, in the sense that it seeks to maximize households' lifetime utility, (8), subject to its own budget constraint, to the economy's resource constraint, and to the decisions taken by the private sector. The period budget constraint of the government is

$$G_t + T_t + B_t = q_t B_{t+1} + \tau_t(w_t + r_t K_t) + \tau_{c,t} C_t + \tau_{x,t} p_t K_{t+1}. \tag{10}$$

The right-hand side of equation (10) represents government revenues, which are made up of debt issues, $q_t B_{t+1}$, and income from taxation (income, consumption, and investment taxes). The left-hand side is total government expenditure, including the provision of the public good, transfers, and the repayment of outstanding debt. In the next section we characterize optimal, time-consistent fiscal policy in this model economy.

2.4 Markov-perfect Optimal Fiscal Policy

The government is assumed to lack both intra- and inter-period commitment to fiscal policy. This implies that the government can change its policy both retroactively (within the year) and prospectively, thus undermining the credibility of any announcement about current and future fiscal policies.

To characterize time-consistent optimal policy we focus on Markov-perfect equilibria. Because of its lack of *inter-period* commitment, the government must make policy decisions sequentially, foreseeing its future behavior, and that of the households, when choosing policy for the current period. Because of its lack of *intra-period* commitment —and hence of its ability to retroactively change its policy any time within the fiscal fiscal— the government is the last to move. The timing of actions within each period is thus as follows. The households first make their consumption/savings decisions. Household resources that are not consumed are set aside to pay taxes and save. Then, the government sets fiscal policy for the current period. If the income tax rate turns out to be different from the one expected by the households, savings will be distorted. Debt issues will pin down the composition of the savings portfolio between physical capital and government debt.

We next present the maximization problem solved by each agent in turn.

2.4.1 The Maximization Problem of a Typical Household

The household chooses how much to consume and save, and how to allocate savings between physical capital and government debt. In making these decisions, the household must foresee both current and future governments' fiscal policy.

The maximization problem solved by a household that holds physical assets, k, and government debt, b, is now presented. Household expectations on current and future policies are denoted as: *(i)* the tax rate on income is expected to be set as $\psi_\tau : (K \times B) \to \tau$, both in the current and in future periods; *(ii)* debt issues are expected to be set as $\psi_{B'} : (K \times B) \to B'$; *(iii)* government expenditure on the public good is expected to be set according to the policy $\psi_G : (K \times B) \to G$. Denoting the vector of endogenous, aggregate state variables by $S \equiv (K, B)$, the maximization problem of the household is[4]

$$
\begin{aligned}
v(k, b, S) \quad &= \quad \max_{c, k', b'} \left\{ \ln c + \theta \ln \psi_G(S) + \beta \tilde{v}(k', b', S') \right\} \qquad (11) \\
s.t. \quad & \\
(1 + \tau_c)c + (1 + \tau_x)p(S)k' &+ q(S)b' = b + (1 - \psi_\tau(S)) \left[w(S) + r(S)k \right] + T. \qquad (12)
\end{aligned}
$$

The function $\tilde{v}(k', b', S')$ on the right-hand side of the maximization problem in (11) is the continuation value as foreseen by the household. Note that $S' \equiv (K', B')$ is the next period vector of aggregate state variables as foreseen by the household, i.e., the economy-wide stock of physical capital is expected to evolve according to the law of motion $K' = \mathcal{H}(S)$ and debt issues are expected to be $B' = \psi_{B'}(S)$. In the budget constraint, $p(S)$, $q(S)$, $w(S)$ and $r(S)$ are pricing functions. The maximization problem above, along with the representative household assumption, $k = K$ and $b = B$, yields a consumption function, $C(S)$, that satisfies the following two functional equations

$$
\frac{1}{(1 + \tau_c)C(S)} = \beta \left[\frac{1}{(1 + \tau_c')C(S')} \times \frac{(1 - \psi_\tau(S'))r(S')}{(1 + \tau_x)p(S)} \right] \qquad (13)
$$

and

$$
\frac{q(S)}{(1 + \tau_c)C(S)} = \beta \left[\frac{1}{(1 + \tau_c')C(S')} \right]. \qquad (14)
$$

Equation (13) is the Euler equation and (14) is the standard pricing equation of a claim to one unit of consumption next period.

[4] To simplify notation, we do not make explicit the dependency of the household and the government problems on the exogenous state variables (transfers and tax rates on consumption and investment). It will be made clear below, though, that the policy functions for the income tax rate and debt issues in a Markov-perfect equilibrium depend on these exogenous state variables.

2.4.2 The Maximization Problem of the Government

The period-t government sets the income tax rate for the period, τ, debt issues, B', and the level of expenditure on the public good, G, foreseeing the fiscal policy to be set by successive governments, and taking household consumption, $C(S)$, as given. The government is benevolent and seeks to maximize the household's lifetime utility. The problem of the period-t government is written as

$$V(S) = \max_{\tau,B',G} \left\{ \ln C(S) + \theta \ln G + \beta \tilde{V}(S') \right\} \quad (15)$$

s.t.

$$C(S) + X + G = AK^{\alpha} \quad (16)$$

$$G + T + B = q(S)B' + \tau \left[w(S) + r(S)K \right] + \tau_c C(S) + \tau_x p(S)K' \quad (17)$$

$$K' = DX^{\lambda}K^{1-\lambda} \quad (18)$$

pricing equations (5), (6), (7) and (14),

where $\tilde{V}(S')$ is the continuation value as foreseen by the period-t government. Restrictions $(16)-(18)$ are, respectively, the economy's resource constraint, the government budget constraint, and the capital production technology. We also impose that the fiscal policy set by the period-t government be sustainable, given the expected policy of future governments. Some comments on the maximization problem above are in order. First, from the viewpoint of the period-t government, its fiscal policy distorts neither current consumption nor the total production of the homogeneous good. Further, the government of period t overlooks the distortions that the income tax rate of period t has on consumption and investment in period $t-1$. Second, when choosing the level of expenditure on the public good, G, the period-t government trades off the implied increase in utility from the public good against a reduction in investment in physical capital (public expenditure crowds out investment). When choosing the mix of income taxes and debt issues to finance public spending (the public good, transfers and the repayment of debt), the government must be indifferent between the two sources of revenue.

The following two generalized Euler equations characterize the optimal policy of the period-t government

$$\frac{\theta}{G(S)} \frac{1}{\lambda DX(S)^{\lambda-1}K^{1-\lambda}} = \beta \left[\frac{C_K(S')}{C(S')} + \frac{\theta}{G(S')} \left(\alpha A(K')^{\alpha-1} - C_K(S') + \frac{1-\lambda}{\lambda} \frac{X(S')}{K'} \right) \right] \quad (19)$$

and

$$C_B(S') \left(\frac{1}{C(S')} - \frac{\theta}{G(S')} \right) = 0, \quad (20)$$

where C_K and C_B denote the derivatives of the consumption function, C, with respect to K and B, respectively. (See the Appendix for a derivation of these generalized Euler equations.)

Generalized Euler equation (19) establishes that the marginal value of spending in the public good must equal the marginal value of investment in physical capital. This equation determines the optimal level of government consumption, G, given the households' consumption/savings decision. Generalized Euler equation (20) is a no-arbitrage condition between taxation and debt, establishing that the government must be indifferent between using taxes or debt to finance the last unit of expenditure on the public good. In addition to these generalized Euler equations, fiscal policy must be sustainable, which in our setting implies that debt must be bounded in the long run.

2.4.3 Markov-perfect Equilibrium

We now formally define a Markov-perfect equilibrium in this economy.

DEFINITION: *A Markov-perfect equilibrium is a list of policy functions $C(K,B)$, $\psi_\tau(K,B)$, $\psi_{B'}(K,B)$ and $\psi_G(K,B)$; a value function $\tilde{V}(K,B)$; and pricing functions $p(K,B)$, $q(K,B)$, $r(K,B)$ and $w(K)$ such that:*

(i) Given ψ_τ, $\psi_{B'}$, ψ_G and the pricing functions, the function C solves the household's maximization problem.

(ii) Given C and $\tilde{V}$, functions ψ_τ, $\psi_{B'}$ and ψ_G solve the government's maximization problem.

(iii) The pricing functions are given by (5), (6), (7) and (14)

(iv) $\tilde{V}$ is the value function of the government, that is,

$$\tilde{V}(K,B) = \ln C(K,B) + \theta \ln \psi_G(K,B) + \beta \tilde{V}[DX(K,B)^\lambda K^{1-\lambda}, \psi_{B'}(K,B)],$$

where $X(K,B) = AK^\alpha - \psi_G(K,B) - C(K,B)$.

Inspection of Generalized Euler equations (19) and (20) offers the following insights on the configuration of Markov-perfect equilibrium. First, as explained above, equation (19) is the government's policy function for G. Since (19) depends on the household consumption function, C, and this, in turn, depends on the expected value of G, there are strategic interactions between the decision problems of the household and the government. As it has been amply established in the literature, strategic interactions may lead to multiple equilibria. Second, equation (20) gives the optimal mix of income taxes and debt issues to fund expenditures G. If government debt is not household net wealth in equilibrium, i.e. $C_B = 0$, then equation (20) holds trivially, yielding an indeterminacy between taxes and debt.[5]

The Proposition below presents policy functions in Markov-perfect equilibrium and shows the existence of a multiplicity of such equilibria.

PROPOSITION 1 *There exists a continuous multiplicity of Markov-perfect equilibria in our model economy. Any quadruplet of policy functions in the following family indexed by $a \in (0,1)$ conforms an interior Markov-perfect equilibrium, provided all variables fall within their feasible ranges:*

$$C(K;a) = aAK^{\alpha} \tag{21}$$

$$\psi_{\tau}(a) = 1 - \frac{(1+\tau_{c,t})(1+\tau_{x,t-1})a_X(a)}{\beta(1+\tau_{c,t-1})[\alpha\lambda + (1-\lambda)a_X(a)]} \tag{22}$$

$$\psi_{B'}(K,B;a) = \left(h_1 \frac{B}{AK} + h_2\right)(Aa_X(a))^{\alpha\lambda} D^{\alpha} AK^{\alpha(1-(1-\alpha)\lambda)} \tag{23}$$

$$\psi_G(K;a) = \frac{(1-a)\theta(1-\beta[1-(1-\alpha)\lambda])}{\theta[1-\beta(1-\lambda)]+\beta\alpha\lambda} AK^{\alpha}, \tag{24}$$

where h_1 and h_2 are functions of a and of the exogenous transfers and tax rates on consumption and investment; and $a_X(a)$ is a function of a.

Functions h_1, h_2 and $a_X(a)$, and the proof of the proposition, are shown in the Appendix. Using these policy functions, we now characterize the government consumption-GDP ratios along the equilibrium path of a Markov-perfect equilibrium.

COROLLARY 2 *Along the equilibrium path of a Markov-perfect equilibrium, government expenditure on the public good as a share of GDP, $\{g_t\}$, is given by*

$$g_t = \frac{a_G(a)}{1 + \frac{1-\lambda}{\lambda} a_X(a)} \quad \text{for} \quad t \geq 0, \tag{25}$$

where $a_G(a)$ is a function of a, which is shown in the Appendix.

[5] For a more detailed analysis of the mechanism generating a multiplicity of equilibria under retroactive tax reforms, see Ortigueira and Pereira (2017).

The proof of the Corollary is straightforward. Government expenditure as a share of GDP is obtained from dividing (24) by GDP, which is given by $\left(1 + \frac{1-\lambda}{\lambda} a_X(a)\right) AK^{\alpha}$ along a Markov-perfect equilibrium.

The Proposition above shows the existence of a continuous multiplicity of (expectation-driven) Markov-perfect equilibria indexed by a. That is, whatever expectation the household may have on government policy, i.e. whatever the value for a is chosen by the household, it will become self-fulfilled in equilibrium by the policies set by the government. Hence, any value of $a \in (0, 1)$ that renders economic variables within their feasible ranges conforms a Markov-perfect equilibrium, with policy functions as given in the Proposition.

In addition to equilibria where a remains constant along the equilibrium path, equilibria with time-varying levels of a can be constructed. These are the equilibria we will look at in this paper. As in a strand of the business cycle literature studying models that display equilibrium indeterminacy (see, for example, Benhabib and Farmer 1994), fluctuations in economic variables are generated by randomizations over the set of certainty equilibria. A sunspot variable is assumed to coordinate agents expectations on the value of a_t. Further, agents are assumed to form adaptive expectations on the next-period value, a_{t+1}. That is, the value for a_{t+1} as expected in period t, say a^e_{t+1}, is given by the realized value at t, a_t. By assuming adaptive expectations we preserve the closed-form solutions obtained above.[6] We provide more details on our approach below and plot expectation shocks along the selected equilibrium path. These shocks are presented as $(a_{t+1} - a^e_{t+1})/a_{t+1}$, and give a measure of the "mistakes" made by economic agents.

We then use U.S. data on household consumption expenditure as a share of GDP from 1960 to 2010 to construct a sequence $\{a_t\}$. With this sequence we generate optimal income tax rates and optimal government expenditures as a share of GDP from equations (22) and (25), respectively. This optimal fiscal policy is then compared to U.S. fiscal policy from 1960 to 2010. The next section constructs the fiscal policy variables in the U.S. economy.

3 U.S. Taxation and Government Expenditure from 1960 to 2010

We use data from the U.S. Bureau of Economic Analysis' NIPA Tables to construct fiscal policy variables for the period 1960 − 2010. As pointed out by Prescott (2004), when using models where the households pay all the taxes, national income accounts must be adjusted to

[6] As an alternative to adaptive expectations, we could have assumed a stochastic process for a_t, which the agents would use to take expectations on future values.

render measured variables consistent with the model variables. The first adjustment calls for the removal of *Taxes on Production and Imports* (TPI) net of *Subsidies to Production* (SUB) from *Gross Domestic Product* (GDP). Both TPI and SUB are available from NIPA Tables 3.5 and 3.13, respectively. The resulting adjusted value of GDP corresponds to output in the model ($Y \equiv C + G + pK'$). That is

$$Y = GDP - (TPI - SUB). \tag{26}$$

Since TPI includes consumption and investment taxes as well as property taxes, the adjustment of the components of GDP by expense (consumption, investment and government expenditure) is as follows. Private consumption expenditure is adjusted by net taxes on consumption and by a fraction of property taxes. Similarly, investment expenditure is adjusted by net taxes on investment and by a fraction of property taxes. Finally, government consumption expenditure is adjusted by a fraction of property taxes. The fraction of the property taxes deducted from each component is set equal to the contribution of the component to GDP gross of the property taxes.

3.1 Consumption and Investment Tax Rates

Our construction of the tax rates on consumption and investment follows closely McDaniel (2011), who builds on Prescott (2004). The two starting aggregates to pin down consumption and investment tax rates are *Taxes on Production and Imports* (TPI) and *Subsidies to Production* (SUB). As explained above, besides consumption and investment taxes, TPI also includes property taxes paid both by households and by other entities. Since property taxes paid by households are mostly taxes on owner occupied housing services, these taxes can be thought of as consumption taxes. However, property taxes paid by other entities must be removed from TPI, to obtain $\overline{TPI}$. Then, taxes paid on consumption and investment, net of subsidies, amount to $\overline{TPI} - SUB$. To split this total between taxes on consumption and taxes on investment, Prescott (2004) assumes that two-thirds fall directly on private consumption expenditures and the remaining one-third is distributed evenly over private consumption and private investment. McDaniel (2011), instead, identifies taxes that fall strictly on consumption expenditures and lowers the two-thirds assumed by Prescott (2004) to 0.506, which yields net taxes paid on consumption, say TPI_c, as

$$TPI_c = \left(0.506 + 0.494 \frac{C}{C+I}\right)(\overline{TPI} - SUB),$$

where C denotes private consumption expenditure, and I private investment expenditure, both gross of taxes as reported in the national accounts. Tax rates on consumption are hence constructed as

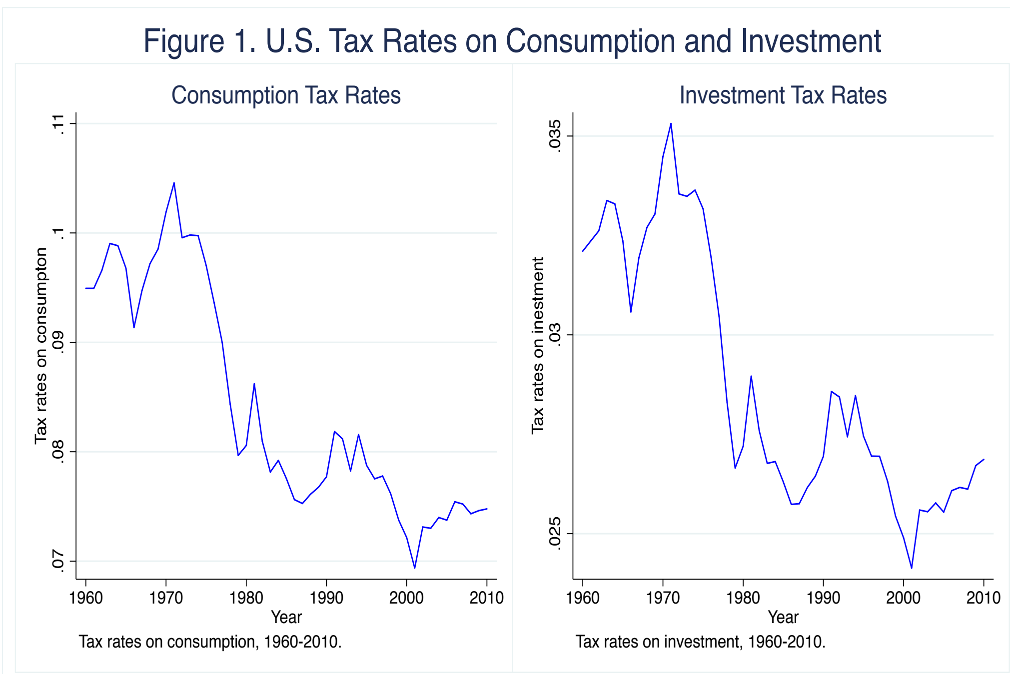
$$\tau_c = \frac{TPI_c}{C - TPI_c}. \tag{27}$$

The left panel of Figure 1 presents the so constructed consumption tax rates for the U.S. from 1960 to 2010.

Tax rates on investment are constructed as

$$\tau_x = \frac{TPI_x}{I - TPI_x}, \tag{28}$$

where TPI_x is revenue from investment taxes, which is given by $TPI_x = \overline{TPI} - SUB - TPI_c$. The right panel of Figure 1 presents the so constructed tax rates on investment from 1960 to 2010.



3.2 Income Tax Rates

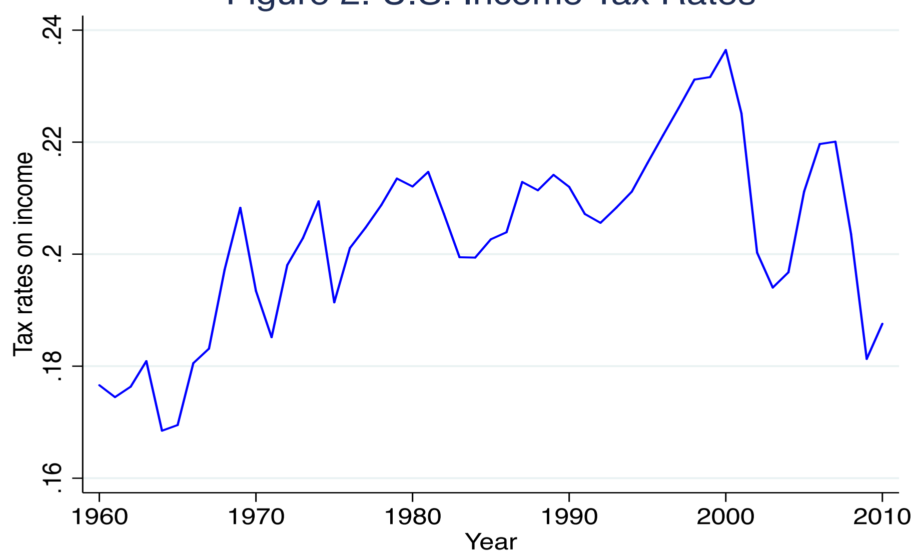
We now construct the empirical counterpart of the income tax rate in our model economy. Since in our model *all* taxes are paid by the households, our construction of the empirical income tax rate includes: (i) *Personal Current Taxes*, PCT, (taxes paid by persons on income); (ii) *Taxes on Corporate Income*, TCI, (taxes paid by firms on income); and (iii) *Contributions for*

Government Social Insurance, CSI, (employers contributions for government social insurance as well as payments by employees). These tax aggregates are available from NIPA Table 3.10. We construct income tax rates by dividing the sum of these three tax aggregates by total income, that is

$$\tau = \frac{PCT + TCI + CSI}{GDP - (TPI - SUB)}. \tag{29}$$

It should be noted that by adding up these three tax aggregates, which actually have different tax bases, we are constructing synthetic direct tax rates in terms of total income. Figure 2 presents the so constructed income tax rates for the U.S. economy from 1960 to 2010.

Figure 2. U.S. Income Tax Rates



Tax rates on income, 1960-2010.

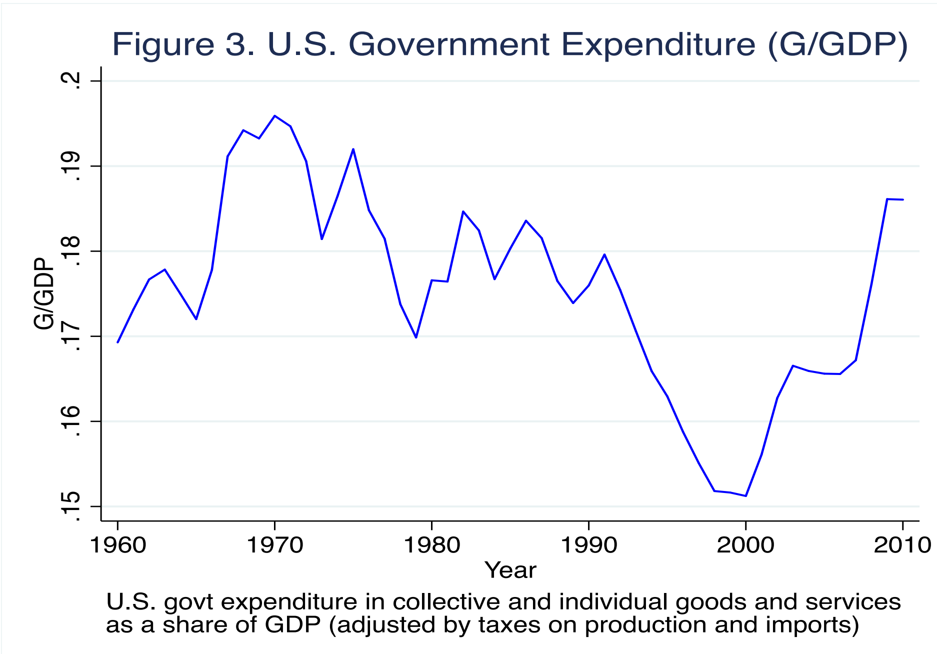
3.3 Government Expenditure on Public Goods and Services

As expenditures on public goods and services, we consider expenditures incurred by the general government both on individual consumption goods and services, and on collective consumption services. Individual consumption goods and services include education, health care, recreation and culture, etc. Collective consumption services include national defense and public order and safety (police, fire, law courts and prisons). Expenditures in these two categories of goods and services appear in NIPA Table 3.9.5 as *Government Consumption Expenditures.* To construct the empirical counterpart of the model's government consumption-to-GDP ratio as we proceed as explained above. We remove a fraction of the property taxes paid by entities other than households from government consumption expenditures in the data, and then divide it by $(TPI-$

SUB)−adjusted GDP. The fraction is chosen to be the contribution of government consumption expenditures (GCE) to GDP gross of the property taxes.[7] Simple algebra yields this empirical counterpart (g) as

$$g = \frac{GCE}{GDP - (\overline{TPI} - SUB)}. \tag{30}$$

Figure 3 below plots this variable from 1960 to 2010.



4 Optimal versus U.S. Policy

In this section we use our model economy to generate optimal income tax rates and government consumption-to-GDP ratios, which we then compare with their empirical counterparts constructed above. We start by calibrating the parameters of our model so that it matches the average values of key variables of the U.S. economy.

4.1 Calibration

We calibrate our model economy using annual U.S. data from 1960 to 2010. There are six parameters in our model: A, λ, D, β, α and θ. Since there is a multiplicity of Markov-perfect

[7] I.e., the empirical counterpart is given by the expression $\frac{GCE-(TPI-\overline{TPI})\times\frac{GCE}{GDP-(\overline{TPI}-SUB)}}{GDP-(TPI-SUB)}$.

equilibria, and a family of equilibrium policy functions indexed by $a \in (0, 1)$, we must also set the value of a. The six parameter values and the value of a are pinned down jointly, so that the corresponding equilibrium matches a set of average values for the 1960-2010 U.S. economy. The value of A is set equal to one. To set the value of λ we use the restrictions in (4), which yield the equivalence between our capital production technology, (2), and the standard law of motion for capital under quadratic adjustment costs, (3). Therefore, the value of λ is set equal to 0.08, which is the annual depreciation rate of capital. The value of parameter D is obtained directly from the value of λ, using (4), as $\lambda^{-\lambda}$. The value of β is set at 0.95, which is the standard value for the annual discount factor used in the macro literature, and matches a rate of return on capital of 5.3%. The two remaining parameters, α and θ, and the value of a, are set to match the following three targets: (i) An average labor's share of income equal to 0.6136; (ii) An average ratio of investment to gross domestic product of 0.2765, which is the ratio obtained from adding up private and public investment, household consumption of durable goods, and net exports, and then dividing by adjusted GDP. By considering net exports as investment, we follow Prescott and McGrattan (2010); (iii) An average ratio of household consumption of non-durable goods and services to gross domestic product of 0.5591. More explicitly, α, θ and a are the solutions to the following system of three equations

$$\frac{1-\alpha}{1+\frac{1-\lambda}{\lambda}a_X(a)} = 0.6136 \tag{31}$$

$$\frac{a_X(a)}{\lambda+(1-\lambda)a_X(a)} = 0.2765 \tag{32}$$

$$\frac{a}{1+\frac{1-\lambda}{\lambda}a_X(a)} = 0.5591, \tag{33}$$

where, as indicated above, the function $a_X(a)$ is shown in the Appendix.

Table 1 presents our baseline economy.

TABLE 1—BASELINE ECONOMY

Parameter	Value	Target
A	1	normalization
λ	0.0800	capital depreciation rate of 8%
D	1.1954	$\lambda^{-\lambda}$
β	0.9500	rate of return on capital of 5.3%
α	0.1700	labor share of income of 0.6136
θ	6.9787	investment-to-output ratio of 0.2765
a	0.7348	household consumption-to-output ratio of 0.5591

It should be noted from our calibration procedure that the average U.S. government consumption-to-GDP ratio is also matched. In the next subsection we construct equilibrium paths for Markov-perfect optimal policy from (22) and (25) under a time-varying sequence for a.

4.2 Non-stationary Expectations and Equilibrium Selection

We now focus our attention on an equilibrium path with time-varying values of a. As discussed above, the multiplicity of perfect-foresight, Markov-perfect equilibria shown in Proposition 1 opens a channel for non-stationary equilibria, and raises the issue of equilibrium selection so that the model's policy prescriptions can be compared to U.S. policy. Selecting an equilibrium path in our model ultimately amounts to pinning down a sequence $\{a_t\}$, which can then be used to generate income tax rates and government consumption as a share of GDP from (22) and (25). Our equilibrium selection approach uses the annual household consumption-to-GDP ratios observed in the U.S. between 1960 and 2010, along with the parameter values set above, to construct a sequence $\{a_t\}$ so that the equilibrium path of our model matches this ratio year by year. It should be emphasized that we are using information only on household consumption and on GDP, but neither information on income tax rates nor on government consumption is being used to pin down the sequence $\{a_t\}$. We will elaborate more on this below. The procedure described above amounts to solving the following system of 51 equations in the 51 unknowns $\{a_t\}_{t=1960}^{2010}$

$$\frac{a_t}{1+\frac{1-\lambda}{\lambda}a_X(a_t)} = \frac{C_t^{US} - TPI_c}{GDP_t^{US} - (\overline{TPI} - SUB)}, \tag{34}$$

where C_t^{US} denotes U.S. household consumption expenditure on non-durable goods and services and GDP_t^{US} denotes U.S. gross domestic product. Again, note that the denominator is not $(TPI - SUB)-$adjusted GDP, because we have to adjust household consumption expenditure not only by consumption taxes but also by a fraction of the property taxes paid by entities other than households. As we did with government consumption expenditure, this fraction is set equal to the contribution of the component to GDP gross of the property taxes.

Once the sequence $\{a_t\}$ has been obtained, we generate optimal income tax rates and government consumption-to-GDP ratios using the expressions in the Proposition and the Corollary above. For the sake of clarity, we write here these expressions evaluated at the sequence $\{a_t\}$

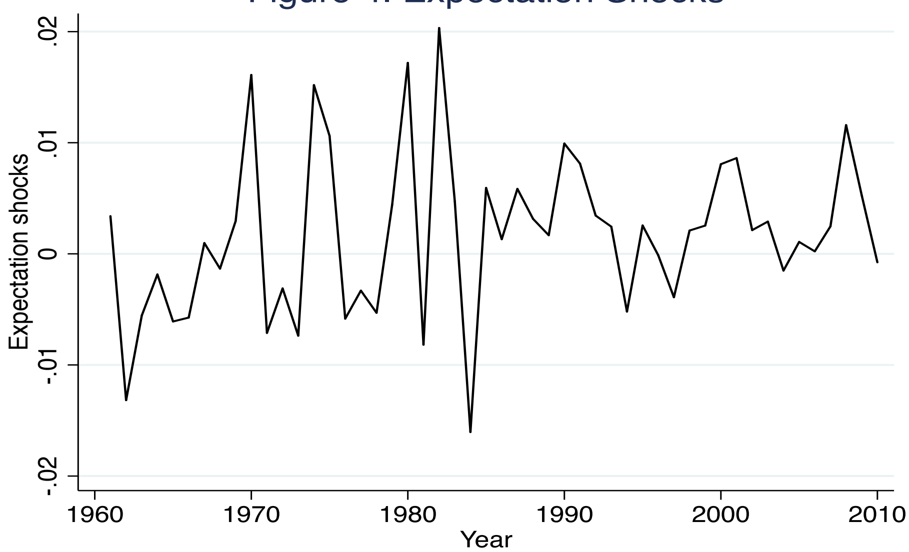
$$\tau_t = 1 - \frac{(1+\tau_{c,t})(1+\tau_{x,t-1})a_X(a_t)}{\beta(1+\tau_{c,t-1})[\alpha\lambda + (1-\lambda)a_X(a_t)]} \tag{35}$$

and

$$g_t = \frac{a_G(a_t)}{1 + \frac{1-\lambda}{\lambda} a_X(a_t)}. \tag{36}$$

Before moving on to the next section where we compare this optimal fiscal policy with the policy observed in the U.S., we briefly discuss expectation shocks along the selected equilibrium. As shown in Figure 4 below, expectation shocks, $(a_{t+1} - a^e_{t+1})/a_{t+1}$, are small (most of them are below 1%), centered at zero, and alternate between positive and negative values. That is, along the equilibrium path agents do not make systematic mistakes by using adaptive expectations.

Figure 4. Expectation Shocks



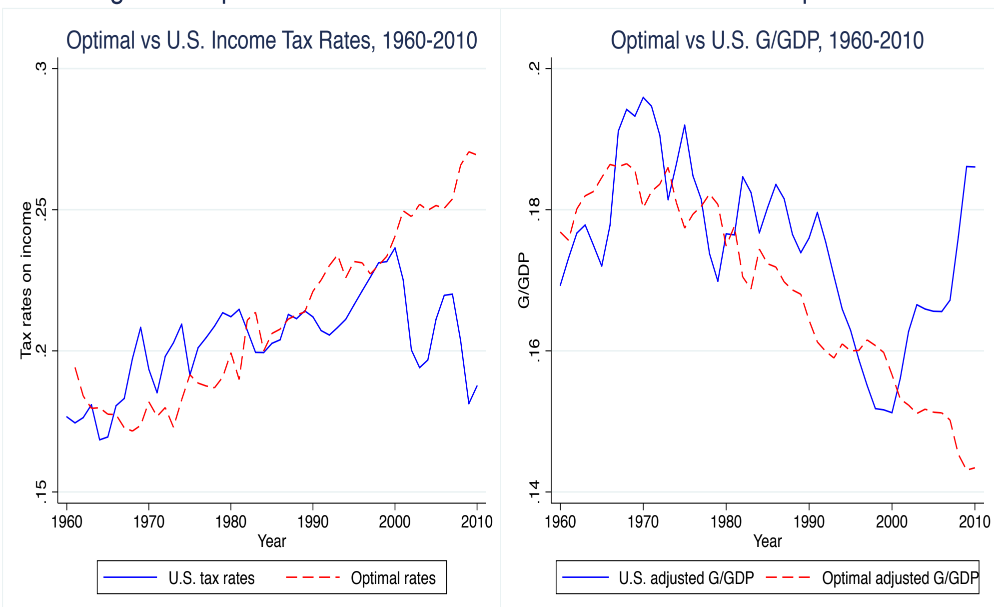
Shocks to expectations along the equilibrium path.

4.3 Comparing Optimal and Actual U.S. Policies

We now compare Markov-perfect optimal policy with U.S. income tax rates and the U.S. government consumption expenditure-to-GDP ratios for the period 1960-2010. The left panel of Figure 5 displays optimal and U.S. income tax rates. From 1960 to 2001, the two rates follow a similar upward trend and are fairly close to each other. However, from 2001 to 2010 optimal and actual tax rates follow opposite trends, opening a gap between the two rates. While the model prescribes that income taxation should have continued to increase during the decade of the 2000s, U.S. income taxes started to decrease in the early 2000s and kept going down for most of the decade.

Regarding government consumption-to-GDP ratios (right panel of Figure 5), the optimal ratios also compare well with the U.S. ratios until the early 2000s. Namely, optimal and U.S. ratios increase during the 1960s; then both start to decline in 1970 and keep on declining until year 2000. However, from 2001 onwards optimal and actual ratios follow opposite trends. While the optimal ratio continues to decline, the U.S. ratio initiates a marked increase until 2010. By way of illustration, the U.S. government consumption-to-GDP ratio was 15.2% in 2001 and 18.6% in 2010. The optimal ratio prescribed by our model is 15.2% in 2001 and 14.2% in 2010.

Figure 5. Optimal vs U.S. Income Taxes and Government Expenditure



4.4 Discussion

Our assessment of the optimality of U.S. income tax rates and government consumption shares must be interpreted correctly within the context of our study. The proposed model economy yields a multiplicity of expectation-driven equilibria, implying that it is not well suited to derive unconditional prescriptions on optimal fiscal policy. However, the model establishes a unique relationship between macroeconomic aggregates and policy variables, which we use to obtain optimal tax rates and government consumption shares conditional on household consumption. That is, using observed household consumption as a primitive, the equilibrium of the model provides the fiscal policy that is optimal given that consumption path. This is exactly our approach to select the equilibrium path that is then compared to actual U.S. fiscal policy. In this sense, the question we answer in this paper is: given the household consumption-to-GDP ratios of the U.S. economy from 1960 to 2010, what are the optimal income tax rates and the optimal government expenditure-to-GDP ratios and how they compare with those of the U.S.?

It is worth noting this approach is useful to assess the optimality of past fiscal policy, but it cannot be used to design future optimal policy. That is, once we know past household consumption, the equilibria of our model informs of the income tax rates and government consumption-to-GDP ratios that would have been optimal given those levels of household consumption. However, our model is unsuited to prescribe future optimal fiscal policy.

The tax cuts of the 2000s. The *Economic Growth and Tax Relief Reconciliation Act of 2001 (EGTRRA),* approved on May 25, 2001 introduced temporal tax cuts totaling 1.6 trillion dollars. In addition to a series of tax rebates, it introduced sizable reductions (between 3 and 5 percentage points) in individual income tax rates as well as in capital gains taxes. The schedule for income tax reductions started on July 1, 2001, with a phase-down period of five years and an expiration date of ten. The *Jobs and Growth Tax Relief Reconciliation Act of 2003 (JGTRRA)* reduced further capital gains taxes and accelerated the reductions in individual income tax rates that had been scheduled in the EGTRRA. According to the tax rates we constructed in Section 3, the combination of EGTRRA and JGTRRA reduced the income tax rate from 23.9% in year 2000 to 19.2% in 2004. The bulk of this reduction came from cuts in personal current taxation and in contributions for government social insurance as shares of adjusted GDP. Since their approval, EGTRRA and JGTRRA have triggered heated debates on the optimality of such tax cuts. A group of academics and the Secretary of the Treasury at the time strongly opposed the cuts, arguing that they would worsen the long-term budget outlook. Others endorsed the cuts but warned that they should be offset with government spending cuts. According to our results, the tax reductions of the early 2000s were non-optimal, given the household consumption-to-GDP ratios at the time.

In February of 2009, the Obama Administration approved the *American Recovery and Reinvestment Act (ARRA)* as an stimulus to consumer spending in the wake of the 2008 Great Recession. By the end of fiscal year 2009, the cut in taxes brought in by ARRA already amounted to \$92.8 billion. Even though our prescribed policy is conditional on observed household consumption, we abstract from fluctuations in hours worked. To the extent that during recessions market work falls and time devoted to leisure increases (Aguiar, Hurst and Karabarbounis 2013), this omission renders our model less suited for assessing fiscal policy at business cycle frequencies. We argue, however, that this caveat is unlikely to affect our assessment of the early 2000s tax cuts, especially the JGTRRA of 2003. The 2001 recession was very short-lived (from March 2001 to November 2001, according to the NBER). While it could be argued that the EGTRRA of May 2001 was a response to the recession, the U.S. economy was already out of the recession when the EGTRRA was approved.

The spending hikes of the 2000s. U.S. government consumption shares increased from 15.1% of adjusted GDP in 2000 to 18.6% in 2010. This increase was the result of both federal and state and local increases in consumption expenditure. At the federal level, both defense and non-defense consumption expenditures increased. For example, expenditure on national defense went from 3.2% of adjusted GDP in 2000 to 4.8% in 2010; federal non-defense consumption expenditure went from 1.75% of adjusted GDP to 2.6%. State and local consumption expenditure increased from 10.2% in 2000 to 11.2% in 2010. According to our model, the spending hikes of the 2000s were non-optimal, given the household consumption-to-GDP rations at the time. Our model prescribes that, given the evolution of the household consumption shares during the first decade of the 2000s, government consumption should had been reduced from 15.5% in 2000 to 14.2% in 2010.

As discussed above, we find it useful to stress that our results should not be used to draw conclusions on the optimality of U.S. fiscal policy at business-cycle frequencies. Since our analysis abstracts from an endogenous labor-leisure choice, our assessment of U.S. fiscal policy should be interpreted at lower-than-business-cycle frequencies and not as policy prescriptions during recessions.

5 Concluding Remarks

This paper provides an assessment of the optimality of U.S. income taxation and government consumption shares from 1960 to 2010. The normative theory used to generate optimal policy builds on the public finance literature initiated by Judd (1985) and Chamley (1986). We relax the assumption of government full commitment to policy and focus our attention on the Markov-perfect equilibria of a model where the government acts sequentially and has the ability to change taxes retroactively. We find a continuous multiplicity of such equilibria, from which we obtain a unique relationship between optimal fiscal policy and household consumption shares. By using this relationship and U.S. data on household consumption shares we generate the fiscal policy that is optimal for those shares of household consumption.

When optimal and actual income tax rates and government consumption shares are compared, our model supports the optimality of U.S. fiscal policy only during the period between 1960 and 2000, but not from 2001 to 2010. In this later period, optimal and actual policies follow opposite trends, with actual income tax rates decreasing below their optimal values and government consumption shares increasing above the levels prescribed by our model.

APPENDIX

PROOF OF PROPOSITION 1:

Markov-perfect equilibrium policies are derived using a guess-and-verify approach. This approach proceeds in three steps. In the first step, we conjecture parametric forms for the equilibrium consumption function, C, and the continuation value function, $\tilde{V}$, and derive the government's policy function for the provision of the public good, ψ_G. In the second step, we use the government's budget constraint and the policy function ψ_G, together with a no-Ponzi scheme constraint, to derive the policy function $\psi_{B'}$, conditional on the functions conjectured in the first step. Then, we use the household Euler equation, the debt sustainability condition and the restriction that the continuation value function must solve the government Bellman equation [i.e., $V = \tilde{V}$)] to derive the tax policy function, ψ_τ, and the parameters in the conjectured functions C and $\tilde{V}$.

Conjectures. We conjecture that household consumption is of the form $C = aAK^\alpha$, where a is a parameter to be determined. The continuation value for the period-t government is conjectured to be of the form $\tilde{V}(K') = A_1 + A_2 \ln K'$, where A_1 and A_2 are parameters to be determined.

The period-t government's maximization problem. The government chooses fiscal policy for the current period, subject to the resource constraint, to its budget constraint, to the no-arbitrage condition between physical capital and public debt, equation (14), to the household consumption function, and to the pricing functions for p, r and w.

Plugging the pricing functions into the period-t government's maximization problem under the conjectures above, this problem becomes

$$\max_{B',\tau,G} \{\ln(aAK^\alpha) + \theta \ln G + \beta(A_1 + A_2 \ln K')\}$$

$$s.t.$$

$$aAK^\alpha + X + G = AK^\alpha \tag{37}$$

$$G + T + B = \beta\left(\frac{1+\tau_c}{1+\tau_c'}\frac{K^{\alpha\lambda}}{X^{\alpha\lambda}}\right)B' + \tau\left(AK^\alpha + \frac{1-\lambda}{\lambda}X\right) + \tau_c aAK^\alpha + \tau_x\frac{X}{\lambda} \tag{38}$$

$$K' = DX^\lambda K^{1-\lambda}, \tag{39}$$

where equation (37) is the resource constraint, equation (38) is the government budget constraint, and equation (39) is the production technology in the capital good sector.

The first-order condition of the government maximization problem with respect to expenditure on the public good is

$$\frac{\theta}{G} = \beta A_2 \frac{\lambda D X^{\lambda-1} K^{1-\lambda}}{K'}, \tag{40}$$

which, after plugging the value for K' from the capital production technology, it yields

$$\frac{\theta}{G} = \beta A_2 \frac{\lambda}{X}. \tag{41}$$

The combination of this first-order condition with the resource constraint, (37), yields the level of government spending in the public good as

$$G = \frac{(1-a)\theta}{\theta + \beta A_2 \lambda} A K^{\alpha}. \tag{42}$$

For future reference, we denote the constant multiplying AK^{α} on the right-hand side of this equation as $a_G(a)$, i.e.

$$a_G(a) \equiv \frac{(1-a)\theta}{\theta + \beta A_2 \lambda}. \tag{43}$$

The level of household spending in the capital good (savings in the physical asset) is

$$pK' = \frac{(1-a)\beta A_2}{\theta + \beta A_2 \lambda} A K^{\alpha}. \tag{44}$$

And the amount of the homogenous good used as an input in the production of the capital good is

$$X = \frac{(1-a)\beta A_2 \lambda}{\theta + \beta A_2 \lambda} A K^{\alpha}. \tag{45}$$

For future reference, we denote the constant multiplying AK^{α} on the right-hand side of this equation as $a_X(a)$, i.e.

$$a_X(a) \equiv \frac{(1-a)\beta A_2 \lambda}{\theta + \beta A_2 \lambda}. \tag{46}$$

The household Euler equation. Under the consumption function conjectured above the household Euler equation becomes

$$\frac{1}{(1+\tau_c) a A K^{\alpha}} = \frac{\beta(1-\tau')\lambda D X^{\lambda-1} K^{1-\lambda}}{(1+\tau_c') a A K'^{\alpha} (1+\tau_x)} \left(\alpha A K'^{\alpha-1} + \frac{(1-\lambda) X'^{\lambda} K'^{-\lambda}}{\lambda X'^{\lambda-1} K'^{1-\lambda}} \right).$$

After some algebra we get

$$\frac{1}{(1+\tau_c) A K^{\alpha}} = \frac{\beta(1-\tau')\lambda D X^{\lambda-1} K^{1-\lambda}}{(1+\tau_c') A K'^{\alpha} (1+\tau_x)} \left(\alpha A K'^{\alpha-1} + \frac{(1-\lambda)}{\lambda} \frac{X'}{K'} \right).$$

Using the capital production technology and the policy function for X, equation (45), we get

$$\frac{1+\tau_x}{1+\tau_c} = \beta \left(\frac{\alpha\lambda}{a_X(a)} + 1 - \lambda \right) \left(\frac{1-\tau'}{1+\tau_c'} \right).$$

We then obtain the tax rate on income in period $t+1$ as

$$\tau' = 1 - \frac{(1+\tau_c')(1+\tau_x)a_X(a)}{\beta(1+\tau_c)[\alpha\lambda + (1-\lambda)a_X(a)]}. \tag{47}$$

Debt and tax policy under debt sustainability. We now use the debt sustainability requirement to derive the debt and tax policy functions. For the sake of expositional clarity we will use here time subscripts to date variables. We will return to our previous notation when there is no risk of ambiguity.

The period-$(t+1)$ government budget constraint is

$$G_{t+1} + T_{t+1} + B_{t+1} = q_{t+1}B_{t+2} + \tau_{t+1}\left(w_{t+1} + r_{t+1}K_{t+1}\right) + \tau_{c,t+1}C_{t+1} + \tau_{x,t+1}p_{t+1}K_{t+2}. \tag{48}$$

Using the pricing equations, (5), (6), (7) and (14), the policy function for government expenditure on the public good, (42), and the policy function for the amount of the homogeneous good used as an input into the production of the capital good, (45), this budget constraint becomes

$$\begin{aligned} a_G(a)AK_{t+1}^{\alpha} + T_{t+1} \quad &+ \quad B_{t+1} = \beta\frac{1+\tau_{c,t+1}}{1+\tau_{c,t+2}}\frac{AK_{t+1}^{\alpha}}{AK_{t+2}^{\alpha}}B_{t+2} + \\ &+ \quad \tau_{t+1}\left(AK_{t+1}^{\alpha} + \frac{1-\lambda}{\lambda}a_X(a)AK_{t+1}^{\alpha}\right) + \left(a\tau_{c,t+1} + \frac{a_X(a)}{\lambda}\tau_{x,t+1}\right)AK_{t+1}^{\alpha}. \end{aligned} \tag{49}$$

As indicated above, transfers to households and tax rates on consumption and investment are assumed to be exogenously given. We write transfers as a fraction, $a_{T,t}$, of production of the homogeneous good, i.e. $T_t = a_{T,t}AK_t^{\alpha}$. Plugging this expression for transfers into the equation above and dividing both sides of this equation by AK_{t+1}^{α} it yields

$$\begin{aligned} a_G(a) + a_{T,t+1} + \frac{B_{t+1}}{AK_{t+1}^{\alpha}} \quad &= \quad \beta\frac{1+\tau_{c,t+1}}{1+\tau_{c,t+2}}\frac{B_{t+2}}{AK_{t+2}^{\alpha}} \\ &+ \quad \tau_{t+1}\left(1 + \frac{1-\lambda}{\lambda}a_X\right) + a\tau_{c,t+1} + \frac{a_X(a)}{\lambda}\tau_{x,t+1}. \end{aligned} \tag{50}$$

Rearranging, we obtain

$$\begin{aligned} \frac{B_{t+2}}{AK_{t+2}^{\alpha}} \quad &= \quad \frac{1}{\beta}\frac{1+\tau_{c,t+2}}{1+\tau_{c,t+1}}\frac{B_{t+1}}{AK_{t+1}^{\alpha}} \\ &+ \quad \left(a_G(a) + a_{T,t+1} - \tau_{t+1}\left(1 + \frac{1-\lambda}{\lambda}a_X\right) - a\tau_{c,t+1} - \frac{a_X(a)}{\lambda}\tau_{x,t+1}\right)\frac{1}{\beta}\frac{1+\tau_{c,t+2}}{1+\tau_{c,t+1}} \end{aligned} \tag{51}$$

Note that τ_{t+1} is given by (47) as a function of $\tau_{c,t}, \tau_{c,t+1}, \tau_{x,t}$ and a (the parameter in the conjectured consumption function). For clarity of exposition, let us introduce the following notation. We denote the term multiplying $\frac{B_{t+1}}{AK_{t+1}^{\alpha}}$ on the right-hand side of equation (51) by $h_{1,t+1}$, i.e.

$$h_{1,t+1} \equiv \frac{1}{\beta}\frac{1+\tau_{c,t+2}}{1+\tau_{c,t+1}}. \tag{52}$$

The second addend on the right-hand side is denoted by $h_{2,t+1}$, i.e.,

$$h_{2,t+1} \equiv \left(a_G(a) + a_{T,t+1} - \tau_{t+1} \left(1 + \frac{1-\lambda}{\lambda} a_X(a) \right) - a\tau_{c,t+1} - \frac{a_X(a)}{\lambda} \tau_{x,t+1} \right) h_{1,t+1}, \tag{53}$$

where τ_{t+1} is given by (47). It should be noted that both $h_{1,t+1}$ and $h_{2,t+1}$ are hence determined by exogenous variables (transfers and tax rates on consumption and investment).

With this notation, equation (51) is written as

$$\frac{B_{t+2}}{AK_{t+2}^{\alpha}} = h_{1,t+1} \frac{B_{t+1}}{AK_{t+1}^{\alpha}} + h_{2,t+1}. \tag{54}$$

Hence, in a Markov-perfect equilibrium we must have that

$$\frac{B_{t+1}}{AK_{t+1}^{\alpha}} = h_{1,t} \frac{B_t}{AK_t^{\alpha}} + h_{2,t}, \qquad \text{for all } t \geq 0. \tag{55}$$

Using the production technology in the capital good sector, the policy function for X, and adopting our notation in the paper, the debt policy function in a Markov-perfect equilibrium is given by

$$B' = \left(h_1 \frac{B}{AK^{\alpha}} + h_2 \right) (Aa_X(a))^{\alpha\lambda} D^{\alpha} AK^{\alpha(1-(1-\alpha)\lambda)}. \tag{56}$$

It should be noted however that the first-order, non-homogeneous difference equation (55) in the ratio of public debt to production in the consumption-good sector contains unstable solutions. Let us assume that the exogenous sequences $\{\tau_{c,t}, \tau_{x,t}, a_{T,t}\}$ converge in finite time to constant values. As a result, $h_{1,t}$ and $h_{2,t}$ converge to constant values, say $\hat{h}_1$ and $\hat{h}_2$, at some large t. In particular, $\frac{1+\tau_{c,t+2}}{1+\tau_{c,t+1}} = 1$ in the long run and since $\beta < 1$, we get that $h_{1,\infty} > 1$. Hence, we must impose initial conditions B_0 and K_0 such that government debt, given by policy function (55), is sustainable. That is,

$$B_0 = \frac{1}{\prod_{t=0}^{\hat{T}-1} h_{1,t}} \left(\frac{\hat{h}_2}{1-\hat{h}_1} - \sum_{t=0}^{\hat{T}-1} \left(\prod_{j=t+1}^{\hat{T}-1} h_{1,j} \right) h_{2,t} \right) AK_0^{\alpha} \tag{57}$$

where $\hat{T}$ is an arbitrarily large time period.

We now derive the policy function for income taxation, ψ_{τ}. This is obtained from the budget constraint of the period-t government

$$G_t + T_t + B_t = q_t B_{t+1} + \tau_t (w_t + r_t K_t) + \tau_{c,t} C_t + \tau_{x,t} p_t K_{t+1}. \tag{58}$$

Plugging into this budget constraint the conjectured policy function for consumption, the policy functions for government spending in the public good (equation 42), the policy function for the amount of the homogeneous good used as an input in the production of capital (equation 45), and the pricing equations we obtain

$$\begin{aligned} a_G(a)AK_t^\alpha + T_t + B_t &= \beta\frac{1+\tau_{c,t}}{1+\tau_{c,t+1}}\frac{AK_t^\alpha}{AK_{t+1}^\alpha}B_{t+1} \\ &+ \tau_t\left(AK_t^\alpha + \frac{1-\lambda}{\lambda}a_X(a)AK_t^\alpha\right) + \left(a\tau_{c,t} + \frac{a_X(a)}{\lambda}\tau_{x,t}\right)AK_t^\alpha. \end{aligned} \tag{59}$$

Dividing both sides of this equation by AK_t^α, using the debt policy function (equation 55), and rearranging yields the tax rate on household income in period t as

$$\psi_\tau(;a) = 1 - \frac{(1+\tau_{c,t})(1+\tau_{x,t-1})a_X(a)}{\beta(1+\tau_{c,t-1})[\alpha\lambda + (1-\lambda)a_X(a)]}. \tag{60}$$

The conjectured continuation value $\tilde{V}$ solves the government's Bellman equation. We now obtain the parameters in the conjectured value function, $\tilde{V}$, so that it solves the Bellman equation of the government. That is

$$A_1 + A_2\ln K = \ln(aAK^\alpha) + \theta\ln G + \beta\left(A_1 + A_2\ln K'\right),$$

where G and K' are the values obtained above under the conjectures. After plugging the values of G and K' we get

$$A_1 + A_2\ln K = \ln(aAK^\alpha) + \theta\ln(a_G(a)AK^\alpha) + \beta A_1 + \beta A_2\ln\left(D(a_X(a)AK^\alpha)^\lambda K^{1-\lambda}\right).$$

From this equation we solve for A_1 and A_2 and obtain

$$A_1 = \frac{\ln a + (1+\theta)\ln A + \theta\ln a_G(a) + \beta A_2\lambda\left(\frac{\ln D}{\lambda} + \ln a_X(a) + \ln A\right)}{1-\beta} \tag{61}$$

$$A_2 = \frac{(1+\theta)\alpha}{1-\beta(1-(1-\alpha)\lambda)}. \tag{62}$$

Once A_1 and A_2 have been determined, notice that we cannot uniquely pin down a value for a. Any value of a that yields endogenous variables within their feasible ranges conforms a Markov-perfect equilibrium. This equilibrium multiplicy stemms from the fact that public debt is not households' net wealth and hence the government is indifferent between tax and debt financing.

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